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THE "INVISIBLE HAND" OF PIRACY: AN ECONOMIC ANALYSIS OF THE INFORMATION-GOODS SUPPLY CHAIN[1]

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In this paper, we study the economic impact of piracy on the supply chain of information goods. When information goods are sold to consumers via a retailer, in certain situations, a moderate level of piracy seems to have a surprising positive impact on the profits of the manufacturer and the retailer while, at the same time, enhancing consumer welfare. Such a "win–win–win" situation is not only good for the supply chain, but is also beneficial for the overall economy. The economic rationale for this surprising result is rooted in how piracy interacts with double marginalization. We explain this rationale and develop useful insights for management and policy.

Keywords: Piracy, supply chain, retailer, double marginalization, digital goods, profit, welfare

"He intends only his own gain, and he is in this, as in many other cases, led by an ***invisible hand*** *to promote an end which was no part of his intention. Nor is it always the worse for society that it was no part of his intention."*

(Smith 1776, p. 477)

Introduction

Copyright infringements of information goods, more commonly known as *piracy*, have been a long-standing problem in the markets for such goods, and TV shows are certainly no exceptions. For instance, year after year, HBO's popular TV series *Game of Thrones* has earned the dubious distinction of being enthroned the "most pirated" program (Tassi 2014; Van der Sar 2017). Illustrating the severity of the issue, the season four finale, within just 12 hours of original broadcasting, was downloaded 1.5 million times, which was close to 2 petabytes transferred in half a day. Similar trends have continued with seasons five, six, and seven, with piracy gaining steam as the show continually grows in popularity. As of September 2017, season seven witnessed a total of more than one billion pirated downloads. While an upsurge in piracy around the time of the original broadcast is only natural, what is really intriguing is that a steady level of interest for the older episodes and their pirated copies has continued for a long period of time, well after they became available at Amazon's online retail store. Such a high level of piracy owes itself partly to high prices (Karaganis 2011); to watch *Game of Thrones* on

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Amazon Video, one must pay $3.99 per episode or $38.99 for an entire season of 10 episodes; DVD and Blu-ray versions are also available at Amazon for about $30 and $40 per season, respectively.[2]

The context of TV shows is instructive. For old shows, such as seasons 1–6 of *Game of Thrones*, the wholesale–retail vertical structure is clearly visible, with a near-monopolistic manufacturer (HBO) as well as a near-monopolistic retailer (Amazon).[3] As is well-known, such vertical structures may face the issue of *double marginalization*, a vertical externality where the retailer is compelled to set the price higher and reduce the output, compared to what would be desired by a vertically integrated channel (Tirole 1992, p.175). Even for current shows that are on the air, the structure closely resembles a supply chain made up of a wholesaler and a retailer. The cable network (e.g., HBO) charges the local cable operator (e.g., Comcast) a per-subscriber monthly fee akin to the wholesale price (Belleflamme and Peitz 2010, p. 435). Given this wholesale price, the cable operator then decides on its own margin, which writes the final retail price tag. More importantly, "the cable operator enjoys a local monopoly, and the cable network offers a product differentiated from its rivals', so double marginalization is indeed a hazard" (Caves 2005, p. 235).[4] Evidently there are two issues present in this context at the same time—piracy and double marginalization—and given that prior research considers only one or the other, it is difficult to speculate their interplay or to discern how the legal channel or policymakers should view antipiracy efforts in this rather complicated setting. In more general terms, we are curious about vertical structures that face the dual threat of piracy and double marginalization.

In fact, a TV show is just one of several such examples, and many other information goods afflicted with piracy are also sold through retailers; we discuss them in the next section. To capture the realities of these markets, we model a supply chain where a manufacturer first sells to a retailer at a wholesale price, and the retailer then resells to consumers at a retail price. In our vertical structure, both the manufacturer and retailer have monopolistic pricing power. These monopolies, in essence, constitute an abstraction of a vertical structure where both up- and downstream firms have *some* pricing power. Therefore, on top of the challenges posed by piracy, our legal channel must also contend with the issue of double marginalization. Naturally, the following research questions emerge:

- How does double marginalization compound a manufacturer's response to piracy? Does piracy affect both the manufacturer and retailer the same way?
- Does the manufacturer still prefer stricter enforcement, as recommended by prior literature for vertically integrated channels where the manufacturer sells directly to the consumer? How should the retailer then view enforcement efforts against piracy?
- What are the implications of increasing enforcement efforts on consumer and social welfare? Specifically, to what extent does prior research on integrated channels remain applicable?

Answers to these questions reveal a curious interplay between piracy and double marginalization: piracy can actually reduce, or completely eliminate at times, the adverse effect of double marginalization. In other words, we seem to end up in a situation where two wrongs do make a right. Specifically, piracy can have a markedly different impact on the manufacturer from what has been suggested previously: the manufacturer can actually be better off when enforcement is weaker. Surprisingly, at the same time, the retailer can gain, too. What is even more surprising, though, is that this gain in the channel profit need not come at the expense of consumers, and piracy may, in fact, lead to a "win–win–win" situation. Such effects of piracy do remind us of the *invisible hand* of markets. Even when every player acts in his or her own self-interest—the manufacturer and retailer maximizing their respective profits and consumers their own utility—piracy somehow makes every selfish actor richer, resulting in the ultimate benevolent outcome of a higher aggregate surplus.

When tied together with prior work on piracy, our results become even more edifying. As has been pointed out by many researchers, legal measures against piracy are often expensive, and before jumping on any bandwagon of antipiracy movement, one must pause to think whether there could be any undesirable repercussion from piracy enforcement (Chen and Png 2003). As explained in this literature, piracy can inject some "shadow" competition into an otherwise monopolistic market (Lahiri and Dey 2013); although consumers gain from this competition, the manufacturer has not been found to be as fortunate. Curiously, this changes in our setting, where piracy limits the pricing power of not just the manufacturer but also of the retailer. Even though a

[2]Retail prices are as of October 12, 2017. at Amazon.com.

[3]Unlike their physical counterparts, an information good intrinsically lacks close substitutes, which immediately grants some monopoly power to its manufacturer. Retailers also command some degree of pricing power for these products; we will illustrate this point later in Table 1.

[4]The same concern about the existence of double marginalization in the cable TV industry has also been shared by other scholars (e.g., Cabral 2017, p. 336). In fact, in a recent work, Crawford et al. (2017) find significant empirical support for the existence of double marginalization in this industry.

limitation on its own pricing power is not good for the manufacturer, the limitation on the retailer's power surely is; a reduction in the retailer's power means less of an adverse impact from double marginalization. An analogous logic is true for the retailer as well. As in prior literature, consumers continue to be beneficiaries of weakening monopolies, leading to an unexpected win–win–win situation.

Now, since the vertical externality arises primarily out of a lack of competition (Tirole 1992, p. 175)—and since piracy can indeed be viewed as a shadow competition (Lahiri and Dey 2013)—it may be tempting to presume that our setting is a special case of up- or downstream competition. This impression would, however, be false. More upstream (downstream) competition makes the retailer (manufacturer) better off by limiting the manufacturer's (retailer's) pricing power. Piracy, however, not only limits the manufacturer's (retailer's) power but also ties the same hand that feeds the retailer (manufacturer). Thus, there are no win–win–win situations with either up- or downstream competition although that is evidently possible in the context of piracy. Viewed differently, the invisible hand of piracy might work even when the invisible hand of markets fails.

Double Marginalization and Information-Goods Markets

To understand why double marginalization is significant in a broad variety of information-goods markets, note that such goods are primarily sold in four formats:

- **Offline**: Large varieties of music, movies, TV shows, video games, and consumer software are sold offline on discs (CD/Vinyl/DVD/Blu-ray). These packaged goods, for which the retailer maintains a physical inventory, are mostly marketed through a wholesale arrangement where the manufacturer charges a wholesale price per disc but the retailer sets its final price. Even though newer formats are emerging, physical discs are not going away any time soon, and neither is the wholesale model for selling them. For instance, in 2015, the total music album sales in the United States stood at 241.4 million units, out of which compact discs (CD) at 125.6 million units accounted for 52%. In the same year, the total sales of computer and video games in the United States reached an impressive $16.5 billion, with about a half of the sales in the disc format. Even in the movie industry, digital video discs (DVD) including the Blu-ray format—with current annual packaged sales hovering around $6 billion in the United States alone—are predicted to remain popular for the foreseeable future (Morris 2016).

- **Subscription:** Many information goods are sold as monthly subscriptions and are often streamed to consumer devices. Examples include cable TV (such as Comcast), video streaming services (such as Amazon and Netflix), and music (such as Spotify and Amazon). Here as well, the pricing structure frequently resembles the wholesale arrangement. The context of cable TV, where the cable operator pays a per-subscriber fee to the content provider, has already been discussed. Similar licensing agreements exist for streaming services as well. One such example is the much-publicized agreement between Amazon and Epix. Originally signed in 2012 and then renewed in 2015, this agreement grants Amazon exclusive rights—and hence monopoly power—to Epix's content, which includes popular movies such as the *Hunger Games* series.

- **Download**: There is also a growing market for information goods where the digital content can be downloaded for a price, Apple's iTunes store being a prime such example. While Apple typically employs agency-selling—essentially a revenue-sharing mechanism that may not suffer much from double marginalization[5]—much downloadable content is still licensed through a wholesale contract. The story of e-books is quite pertinent here. According to PricewaterhouseCoopers, the U.S. market for e-books, which was about $2.3 billion in 2011, is expected to grow to $8.2 billion by 2017. This huge e-book market is primarily contracted using the wholesale model, Google Play and Amazon Kindle being two prominent examples. Notably, Apple, after adopting the agency model, faced a huge antitrust lawsuit, which they settled for $450 million.

- **OEM Licensing**: Finally, we cannot overlook the software market when discussing the issue of piracy. Microsoft's 10-K filings with the Securities and Exchange Commission reveal that the software giant ships hundreds of millions of licenses of its Windows operating system each year, with the revenue hovering around the $20 billion mark. A big portion, about 75–80% of the total sales, happens through original equipment manufacturer (OEM) licensing, and the rest through retailers. Here, too, the licensing arrangement follows the whole-

[5]The gentle reader is forewarned that revenue-sharing contracts do not always work in practice. Such a contract, for example, does not coordinate the supply chain when demand depends on costly marketing efforts. Thus, although it might be tempting to think that revenue sharing between a video game publisher and a developer would align their incentives, if costly advertising becomes critical to boosting sales, it can be shown that royalty payments based on a revenue-sharing formula would not coordinate the channel (Cachon and Lariviere 2005; Gil and Warzynski 2015). In other words, double marginalization could exist despite revenue sharing.

sale model: OEM licensees and retailers pay a per-copy price to Microsoft, although this price varies from time to time and from one version to another.

Evidently, in vast swaths of information-goods markets (music, movies, TV shows, video games, e-books, and software), the wholesale model remains popular. This popularity is perhaps rooted in the fact that the wholesale model is "administratively cheaper" (Cachon and Lariviere 2005, p. 30).[6]

Not only is the class of retailer-priced information goods considerable, so also is the pricing power of their retailers. Consider, for example, the retail prices of a few information goods shown in Table 1. It is clear from this table that established retailers have significant pricing power over a large range of products, indicating a lack of *perfect* competition; otherwise, such wide price dispersions from one retailer to another would be hard to explain. Such imperfection may be attributable to a number of factors including, but not limited to, retailers' brand names, exclusive contracts with manufacturers, convenient brick-and-mortar presence, sophisticated recommender systems, lock-in mechanisms such as Amazon Prime, and the long-tail effect.[7] All in all, the presence of the wholesale model, as well as some pricing power across the channel, makes the issue of double marginalization quite pertinent for many information goods.

Literature Review

Our work is at the intersection of two streams of literature: double marginalization and digital piracy. The issue of double marginalization has long been recognized by economists (Spengler 1950): when a monopolistic manufacturer sells through a monopolistic retailer, the vertical externality manifests itself as a higher retail price, a lower demand, and a reduced channel profit. Despite extensive literature on this topic (see Cachon 2003; Höhn 2010), its connection to piracy has remained unexplored.

Moving on to the literature on digital piracy, one of its branches challenges the common wisdom that piracy is always detrimental to the manufacturer. In fact, it identifies several situations in which a manufacturer may find it profitable to tolerate or support piracy. One such situation occurs in the presence of a positive network effect that translates illegal usage into a higher willingness to pay for the legal product (Conner and Rumelt 1991). Just as positive network effects alter the incentive to tolerate piracy, negative network effects do too. For example, denying pirates critical security patches can be counterproductive when doing so makes legal users vulnerable to security attacks, lowering their willingness to pay (August and Tunca 2008). Finally, even when there are no network effects, antipiracy measures that directly diminish the utility of the legal product can also create incentives to tolerate piracy (Vernik et al. 2011). Our context is different. We do not consider any factors that directly affect the utility of the legal product, neither do we model network effects that impact its value indirectly. Yet, we find that piracy can favorably impact the legal channel's profit.

Social planners could also have their own reasons to tolerate piracy. For example, Bae and Choi (2006) show that social welfare can actually increase as a result of piracy even when it is harmful to the manufacturer. Lahiri and Dey (2013) further explain why piracy may unexpectedly lead to more innovation and better quality products, again leading to higher welfare. Chen and Png (2003) highlight the tension between private profits and social welfare: in the battle against piracy, the manufacturer always prefers stricter enforcement even though that reduces social welfare. This should not come as a surprise, since piracy offers consumers an alternative and, in that sense, works as a competitor to the legal product. In contrast, we show that, in the presence of a retailer, piracy could even lead to a win–win–win situation in which the manufacturer, retailer, and consumers all gain simultaneously.

There are other interesting studies as well, and they all shed light on a manufacturer's strategy in the presence of piracy. Chellappa and Shivendu (2005) show how the pirated version of a digital good may serve as its product sample, with important implications for its pricing and versioning decisions. Wu and Chen (2008) explain why a manufacturer may offer a lower quality version of its product to combat piracy. Sundararajan (2004) discusses a monopolist's optimal nonlinear

[6]The main complaint against revenue sharing is that of verifiability: the extent to which the manufacturer can verify the revenue reported by the retailer (Cachon and Lariviere 2005). In the context of digital information goods, verifiability becomes an even more pertinent issue, because these products can be copied in a costless manner—as many times as one desires—often making it quite difficult for the upstream agent to track downstream sales properly.

[7]Indeed, the fastest-growing part of Amazon's business is the long tail of its product line. In ordinary terms, these are the products that are rarely available in traditional retail stores, such rarity essentially granting Amazon near-monopoly power over these items. The influence of such a long tail is not at all surprising given that the retail giant stocks an eye-popping 355,499 titles in its online software store, an overwhelming majority—325,917 to be precise—in the disc format for which the wholesale model is still the standard practice (numbers as of June 8, 2016). The long tail in its music store is also impressive; for instance, again as of June 8, 2016, a simple search on "Jethro Tull" at Amazon.com brought up 1,159 results in the disc format alone, containing every album of the band that has ever been published, including special boxed sets, collectors' editions, and concert recordings. For the sake of comparison, the same search at BestBuy.com and Walmart.com found only 98 and 22 items, respectively.

Table 1. Online Prices of Information Goods across Retailers

Category	Product	Retailers		
		Amazon	Best Buy	Walmart
Movie or TV Show	*Captain America: The Winter Soldier* (Blu-ray, 3D)	$26.96	$29.99	$26.96
	Shameless: Season 3 (Blu-ray)	$32.98	$31.99	$27.69
	Games of Thrones: Season 5 (Blu-ray)	$39.99	$39.99	$34.96
Music	*Frozen* Soundtrack 2-Disk Deluxe Edition (CD)	$14.88	$16.99	$14.88
	Hamilton (Original Broadway Cast Recording; 2CD)	$22.39	$24.99	$24.19
	Blackstar by David Bowie (CD)	$11.99	$13.99	$11.18
Video Game	*Destiny: The Taken King* Legendary Ed. (Xbox 360)	$18.69	$49.99	$30.78
	Call of Duty: Black Ops III (PS4)	$36.99	$59.99	$39.88
	Final Fantasy XIV: Heavensward (PC)	$19.99	$19.99	$25.38
Software	Microsoft Office Home and Student 2016 (PC)	$116.95	$149.99	$115.99
	Dragon Naturally Speaking 13 Premium (PC)	$107.99	$199.99	$195.60
	Kaspersky Total Security 2016 (PC; 5 Devices)	$30.08	$99.99	$97.01

Note: Online prices as of June 8, 2016.

price schedule in the presence of piracy. Gopal and Gupta (2010) explain situations in which product bundling may serve as an antidote to piracy. Johar et al. (2012) show that the manufacturer can use the content-delivery speed as a strategic lever in its battle against piracy. Tunca and Wu (2013) show why various forms of piracy may work against one another, surprisingly helping the manufacturer. Kannan et al. (2016) point out why piracy can be a motivation to build buggy products. Jain (2008) shows that piracy can ease competition between manufacturers of information goods. Our contribution to this stream is that we study piracy in the backdrop of double marginalization and explain why piracy or its threat can serve as an unlikely tool for channel coordination.

Finally, we would like to point out that there is a growing branch of information systems literature that discusses supply chains and associated issues. Among earlier works, Premkumar (2003) draws attention to different distribution strategies and supply-chain configurations prevalent in the music industry. Using a model of a digital manufacturer–retailer chain, Chellappa and Shivendu (2003) examine the impact of piracy under different contracts (fixed-fee versus per-copy) and find that retailers prefer a fixed-fee contract where they pay a one-time licensing fee. Khouja and Park (2007) study the effect of piracy on a creator–manufacturer supply chain for digital experience goods and show that the royalty system does not solve the double-marginalization problem. Jeong et al. (2012) further examine the optimality of various contracts commonly used in the context of music distribution. Their analyses of different contracting arrangements between a record label and a retailer reveal that piracy, in general, depresses profits, and that eliminating double marginalization requires a fixed-fee full-transfer contract. In contrast, we show that piracy can itself arrest double marginalization and can increase profits for the channel constituents.

Moving on to more recent works on digital supply chains, Hao and Fan (2014) show that agency selling, instead of coordinating the channel and leading to lower prices, can surprisingly lead to higher prices; this happens when the pricing of the e-reader becomes intertwined with the pricing of e-books. Abhishek et al. (2016) study whether online retailers should use the agency-selling format versus the more conventional wholesale format and show that the decision depends on the level of competition among e-tailers as well as the spillover and cannibalization effects of the electronic channel on the brick-and-mortar channel. The issues considered in these papers are beyond the scope of our research.

Model

We consider a traditional wholesale model where the manufacturer first decides on the wholesale price $w > 0$. Then, the retailer chooses the retail price $p > 0$. Finally, consumers decide whether to buy, pirate, or forgo use. We traverse this time line backward, starting with consumers' decisions.

Consumer Behavior

We assume that consumers are heterogeneous in terms of their valuations for the legal product:

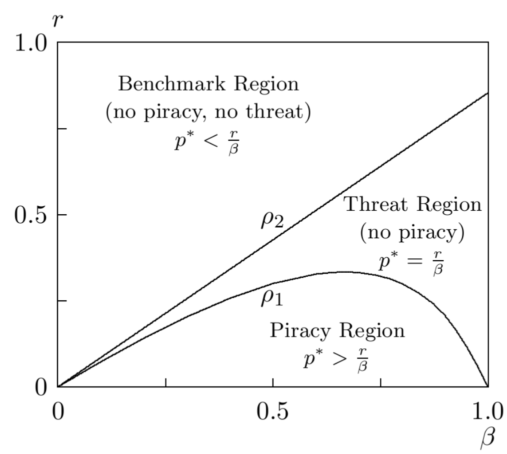

Figure 1. Regions of Piracy

Assumption 1. *Consumers are indexed by their valuation, denoted v, which is uniformly distributed over* [0, 1].

Prior literature typically makes the assumption that the pirated product is inferior to the legal one (e.g., August and Tunca 2008; Bae and Choi 2006; Chen and Png 2003; Jaisingh 2009; Lahiri 2012; Lahiri and Dey 2013; Sundararajan 2004; Tunca and Wu 2013). First, pirated copies usually do not get product support from the manufacturer, as in the case of video games and software for which illegal users are not offered updates and patches. Clearly, illegal copies are of lower quality because they lack newer functionality enhancements and face higher security risks. Second, pirated copies of software or video games often contain embedded malicious codes, and they may even be missing certain important features. Third, examples abound where the physical quality of the pirated product is less than that of the original product (Karaganis 2011). For instance, in the case of pirated movies, pirate sites usually do not have access to fast content delivery networks and often intentionally downgrade the quality of pirated content to ensure a reasonable download time. Even if the physical quality is the same, users of illegal sites typically face a much higher download time. Finally, even when there is little perceptible difference in the physical quality (bit rate) between pirated and legal music files, pirated copies could lack appropriate tags (artist, title, and genre, to name a few); the absence of tags and related indexes makes it difficult to organize and locate these files within a music player. In light of the above, we assume

Assumption 2. *Consumer v gets a value of $v\beta$ from using the pirated version, where $\beta \in (0, 1)$ is the degradation factor of the content.*

On top of having to use content of lesser quality, copyright offenders also face a piracy cost, r, which increases with the level of enforcement against distribution and consumption of pirated goods and, therefore, serves as a proxy for piracy enforcement. It is well recognized in prior literature that anti-piracy measures could impact this piracy cost in two ways: (1) they could impose an acquisition cost of r_1 by making it difficult for consumers to locate or acquire pirated content (see Danaher et al. 2010; Danaher and Smith 2014), or (2) they could inflict an expected legal penalty of r_2 from getting caught (see August and Tunca 2008; Danaher et al. 2010; Danaher et al. 2014; Lahiri and Dey 2013; Tunca and Wu 2013). In our abstraction, r captures both of these costs borne by a consumer, that is, $r = r_1 + r_2$.

Assumption 3. *Consumers face a piracy cost of $r > 0$ which is also a proxy for the level of piracy enforcement.*

Thus, a consumer can enjoy a utility of $(v - p)$ from purchasing the legal version, or $(v\beta - r)$ from a pirated copy. A consumer buys the legal product if the following individual rationality (IR) and incentive compatibility (IC) constraints are satisfied:

$$v - p \geq 0 \Rightarrow v \geq p \qquad \text{(IR-Legal)}$$

and

$$v - p \geq v\beta - r \Rightarrow v \geq \tfrac{p-r}{1-\beta} \qquad \text{(IC-Legal)}$$

Similarly, a consumer would procure a pirated copy if the following constraints are satisfied:

$$v - \tfrac{r}{\beta} \geq 0 \Rightarrow v \geq \tfrac{r}{\beta} \qquad \text{(IR-Pirated)}$$

and

$$v\beta - r > v - p \Rightarrow v < \tfrac{p-r}{1-\beta} \qquad \text{(IC-Pirated)}$$

Two cases are possible: (1) $r \geq \beta$ and (2) $r < \beta$. It is clear that, in the first case, (IR-Pirated) cannot be satisfied for any consumer and piracy disappears trivially, leading to the case we call *benchmark* in Figure 1. Let us now consider the more interesting case of $r < \beta$. Since $\frac{p-r}{1-\beta} > p$ holds only when $p > \frac{r}{\beta}$, the legal and illegal demands for a given $p \in (0, 1)$, respectively denoted $q(p)$ and $\bar{q}(p)$, can be written as

$$q(p) = \begin{cases} 1 - \frac{p-r}{1-\beta}, & \text{if } p > \frac{r}{\beta} \\ 1 - p, & \text{otherwise} \end{cases}$$

and (1)

$$\bar{q}(p) = \begin{cases} \frac{p-r}{1-\beta} - p, & \text{if } p > \frac{r}{\beta} \\ 0, & \text{otherwise} \end{cases}$$

Retailer's Problem

The retailer chooses p in order to maximize its profit $\pi_r(p) = (p - w)\, q(p)$. In a typical manufacturer–retailer setup, where the issue of piracy is absent, the price set by the retailer is expected to be strictly increasing in the wholesale price charged by the manufacturer. However, as it turns out, even this basic intuition does not hold any longer. The threat of piracy might be compelling enough for the retailer to hold the retail price at a fixed value of $\frac{r}{\beta}$ even as the wholesale price changes. We state this curious finding as our first result.

Lemma 1. *The optimal retail price for a given w, $p^*(w)$, is*

$$p^*(w) = \begin{cases} \frac{1-\beta+r+w}{2}, & \text{if } w > \frac{2r}{\beta} - (1-\beta+r) \\ \frac{r}{\beta}, & \text{if } \frac{2r}{\beta} - 1 \leq w \leq \frac{2r}{\beta} - (1-\beta+r) \\ \frac{1+w}{2}, & \text{otherwise} \end{cases}$$

Lemma 1 can be explained as follows. When w is high, the retailer finds it preferable to tolerate some piracy and set p above $\frac{r}{\beta}$. So, the optimal retail price is obtained by maximizing $(p-w)\left(1 - \frac{p-r}{1-\beta}\right)$; see (1). The first order condition immediately leads to the optimal price of $p^*(w) = \frac{1-\beta+r+w}{2}$. Exactly the opposite happens when w is small and the retailer is content with a price below $\frac{r}{\beta}$; the optimal price of $\frac{1+w}{2}$ is then obtained from maximizing $(p - w)(1 - p)$. However, neither of the two solutions above is meaningful when w is moderate. In such a situation, $(p-w)\left(1-\frac{p-r}{1-\beta}\right)$ is decreasing for all $p > \frac{r}{\beta}$, whereas $(p - w)(1 - p)$ is increasing for all $p < \frac{r}{\beta}$. Therefore, it becomes optimal for the retailer to set the price to $\frac{r}{\beta}$, irrespective of the value of w.

Manufacturer's Problem and the Equilibrium

The manufacturer anticipates the retailer's reaction in Lemma 1 and names the optimal wholesale price w^* to maximize $\pi_m(w) = wq(p^*(w))$. Of course, the manufacturer can force the retailer to a retail price of $\frac{r}{\beta}$ by picking a wholesale price in the range $\left[\frac{2r}{\beta} - 1, \frac{2r}{\beta} - (1-\beta+r)\right]$. Clearly, if the manufacturer wishes to do so, it would choose the corner solution $w^* = \frac{2r}{\beta} - (1-\beta+r)$.

Alternatively, the manufacturer may want to confine the retailer to one of the other two regions by choosing an interior solution obtained from the first order condition. Of course, the manufacturer would do so if and only if (1) an appropriate interior solution exists, and (2) the manufacturer's profit from that interior solution is more than the profit from the corner solution. This way, we can determine w^* and obtain the equilibrium retail price, $p^* = p^*(w^*)$.

Proposition 1. (Equilibrium) *Let* $\rho_1 = \frac{3\beta(1-\beta)}{4-3\beta}$ *and* $\rho_2 = \frac{\beta(6-4\beta+\sqrt{2\beta})}{4(2-\beta)}$. *Then,* $\rho_1 < \rho_2$, *and the following equilibrium outcomes emerge:*

- **Piracy Region**: *When* $r < \rho_1$, *both the manufacturer and the retailer find it optimal to tolerate some level of piracy. In this case,* $w^* = \frac{1-\beta+r}{2}$ *and* $p^* = \frac{3(1-\beta+r)}{4}$.
- **Threat Region**: *When* $\rho_1 \leq r < \rho_2$, *there is no piracy, but the threat of piracy affects the pricing decisions. Here,* $w^* = \frac{2r}{\beta} - (1-\beta+r)$ *and* $p^* = \frac{r}{\beta}$.
- **Benchmark Region**: *When* $r \geq \rho_2$, *even the threat of piracy disappears, and the manufacturer and the retailer both behave as if they are in a market not affected by piracy. In this benchmark case,* $w^* = \frac{1}{2}$ *and* $p^* = \frac{3}{4}$.

Figure 1 illustrates the three regions delineated by Proposition 1; the boundaries of these regions, ρ_1 and ρ_2 are also clearly identified as functions of β. Evidently, when enforcement is very high, above ρ_2 to be precise, we end up with $p^* < \frac{r}{\beta}$ in optimality, and neither piracy nor its threat has any effect on the equilibrium. As a result, in the region above ρ_2, the equilibrium prices do not depend on r. In essence, this region is equivalent to a market that is not fraught with piracy in any manner. For this reason, we choose it as our *benchmark* for studying the impact of piracy.

As enforcement falls below ρ_2, the issue of piracy becomes relevant. Specifically, in the region between ρ_1 and ρ_2 in Figure 1, the retail price is always $\frac{r}{\beta}$. This limit price barely keeps piracy away. Although piracy is not present in this region, the threat of piracy is evident from the fact that both p^* and w^* are now directly linked to r and β. Hence, this part of the parameter space is best termed as the *threat region*. Finally, when enforcement becomes even weaker and r falls below ρ_1, piracy surfaces: it is now impossible for the manufacturer, or the retailer, to weed out piracy completely. Accordingly, this region is named the *piracy region*. Note that, when β increases, the pirated product becomes more attractive. Initially, this makes the piracy region wider. However, when β increases further, the pirated product becomes a very close substitute and the legal channel responds by heavily reducing the price of the legal product; this response results in a shrinkage of the piracy region.

A word is now in order on the real-world meaning of Proposition 1. The equilibrium regions identified by it are not just mathematical possibilities; they do provide important practical insights. For example, high prices have often been blamed for high piracy rates in many emerging economies (Karaganis 2011), and it has been suggested that the legal channel should cut prices to combat piracy effectively. What Proposition 1 shows is that this argument is not completely accurate. If enforcement is extremely weak, below ρ_1 in particular, the strategy of eradicating piracy through lower prices is simply futile. There is no way a manufacturer or retailer can outcompete in price an alternative that is nearly free. However, doing so becomes not only viable but also optimal when the cost of the pirated alternative increases beyond a point; this is actually the essence of the threat region, where the pirated good, although forsaken by consumers for all practical purposes, still remains in the hands of a few and continues to pose a credible threat of shadow competition to the legal channel.

Impacts of Piracy

We now carefully analyze the equilibrium outcome in the previous section to investigate the impacts of piracy on the incentives of the manufacturer, the retailer, and consumers.

Manufacturer's and Retailer's Profits

First, we examine whether a reduced level of piracy increases the profits for the manufacturer and retailer. According to prior research, lower enforcement and higher piracy should lead to lower profits (Bae and Choi 2006; Lahiri and Dey 2013). Does this insight still hold for our supply chain?

Recall that, when $r \geq \rho_2$, we are in the benchmark region where piracy is irrelevant and plays no role in the equilibrium. For convenience, we denote the benchmark profits of the manufacturer and retailer by $\pi_{m0} = \frac{1}{8}$ and $\pi_{r0} = \frac{1}{16}$, respectively. The question of interest, therefore, is how the manufacturer's equilibrium profit in the piracy region, or in the threat region, compares to π_{m0}. Likewise, one may ask a similar question about the retailer's profit.

Proposition 2. (Manufacturer's and Retailer's Profits) *Let ρ_1 and ρ_2 be as above. In equilibrium the manufacturer's and retailer's profits are respectively given by*

$$\pi_m^* = \begin{cases} \frac{(1-\beta+r)^2}{8(1-\beta)}, & \text{if } r < \rho_1 \\ \frac{(\beta-r)(r-(1-\beta)(\beta-r))}{\beta^2}, & \text{if } \rho_1 \leq r < \rho_2 \\ \frac{1}{8}, & \text{otherwise} \end{cases}$$

and

$$\pi_r^* = \begin{cases} \frac{(1-\beta+r)^2}{16(1-\beta)}, & \text{if } r < \rho_1 \\ \frac{(1-\beta)(\beta-r)^2}{\beta^2}, & \text{if } \rho_1 \leq r < \rho_2 \\ \frac{1}{16}, & \text{otherwise} \end{cases}$$

Figure 2(a) shows how the equilibrium profits change with enforcement, r. A closer look at these profits, as well as a quick comparison with π_{m0} and π_{r0}, reveals several interesting insights. Somewhat counterintuitively, we find that the impacts of r on these profits are not monotonic. When the enforcement level is low and piracy rampant, the conventional

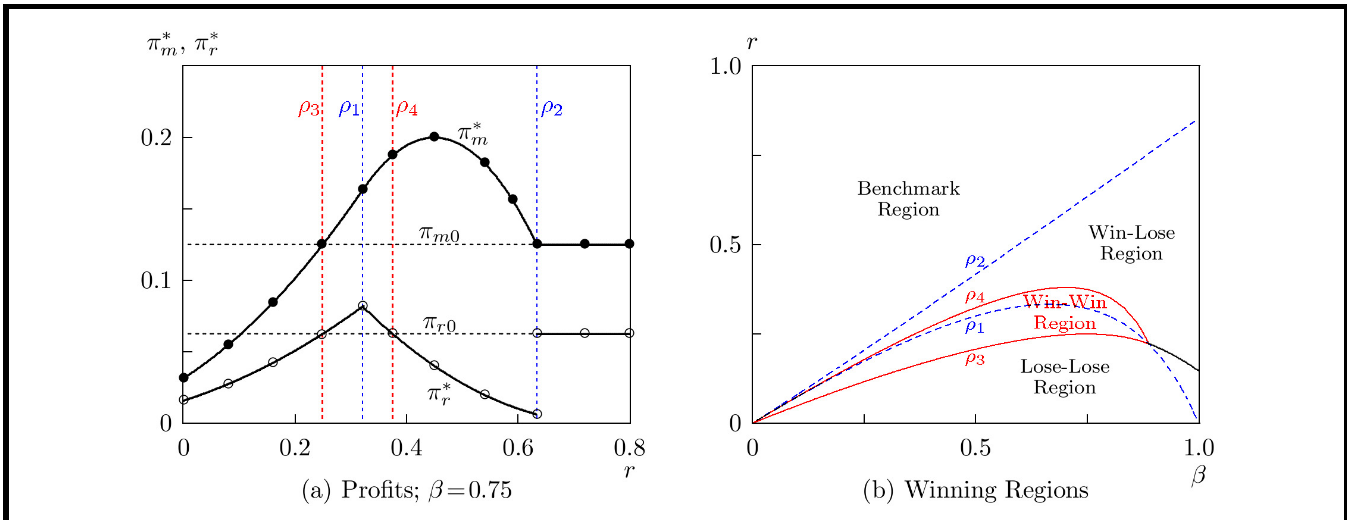

Figure 2. Manufacturer's and Retailer's Profits and Winning Regions

wisdom that the manufacturer's profit increases with r holds, but it need not at higher levels of enforcement. In fact, two new thresholds, ρ_3 and ρ_4 emerge and, as shown in Figure 2(b), the parameter space below $r = \rho_2$ in Figure 1 separates into three new regions:

Theorem 1. (Manufacturer's and Retailer's Incentives) *Let ρ_1 and ρ_2 be as above; also let*

$$\rho_3 = \begin{cases} \sqrt{1-\beta} - (1-\beta), & \text{if } \beta \le \frac{8}{9} \\ \frac{\beta(6-4\beta-\sqrt{2\beta})}{4(2-\beta)}, & \text{otherwise} \end{cases}$$

and

$$\rho_4 = \begin{cases} \beta\left(1 - \frac{1}{4\sqrt{1-\beta}}\right), & \text{if } \beta \le \frac{8}{9} \\ \frac{\beta(6-4\beta-\sqrt{2\beta})}{4(2-\beta)}, & \text{otherwise} \end{cases}$$

Then, for $r < \rho_2$, the following regions emerge:

- **Lose–Lose Region**: *When $r \le \rho_3$, the manufacturer and retailer are both worse off in the presence of piracy than without.*

- **Win–Win Region**: *When $\rho_3 < r < \rho_4$, the manufacturer and retailer are both better off.*

- **Win–Lose Region**: *When $\rho_4 \le r < \rho_3$, the manufacturer is better off, but the retailer is not.*

The most surprising part of Theorem 1 and Figure 2 is the emergence of the win–win region, where the manufacturer and retailer both enjoy higher profits and, therefore, prefer the presence of piracy or its threat. Viewed differently, for both the manufacturer and the retailer, a moderate level of enforcement becomes preferable to a low—or, intriguingly, even a high—level of enforcement. A material implication is that, when situated in this region of moderate piracy, we cannot expect either the manufacturer or retailer to complain too much about ill effects of piracy, nor should we anticipate them to lobby governments and other law-enforcing agencies to step up enforcement efforts. This result is in stark contrast with prior literature, which finds that piracy or its threat can only hurt the manufacturer (Bae and Choi 2006; Lahiri and Dey 2013). Our manufacturer does benefit from piracy, and this benefit is not a consequence of any network effect, neither is it at the expense of the retailer, who benefits as well.

A point to note is that the win–win region is not guaranteed to occur always. Only when $\rho_3 < \rho_4$, equivalently $\beta < \frac{8}{9}$, does this region emerge. The existence of this threshold is instructive, although the value $\frac{8}{9}$ is merely an artifact of the uniform distribution in Assumption 1 and can be higher or lower for other distributions. What it simply tells us is that piracy cannot benefit both the parties when the pirated product is overly competitive. At a large β, the win–win region thus disappears, but interestingly, there is always a win–lose region, where the manufacturer prefers piracy or its threat. This is again in contrast with prior literature that finds the manufacturer's profit to be monotonically increasing in r. Finally, in Figure 2, there is no lose–win region, where the retailer is better off but the manufacturer is not. This is expected since the manufacturer has a first-mover advantage in the sequential game.

Now, when the manufacturer or the retailer, or both, make a higher profit, does it come at the expense of consumer welfare? How should consumers react to higher levels of enforcement? We explore that next.

Price and Consumer Welfare

From Proposition 1, p^* can be expressed as

$$p^* = \begin{cases} \frac{3(1-\beta+r)}{4}, & if\ r < \rho_1 \\ \frac{r}{\beta}, & if\ \rho_1 \le r < \rho_2 \\ p_0 = \frac{3}{4}, & otherwise \end{cases} \tag{2}$$

To understand how consumers may react to piracy, we also consider the consumer surplus

$$CS = \begin{cases} \int_{\frac{p^*-r}{1-\beta}}^{1} (v - p^*)\,dv + \int_{\frac{r}{\beta}}^{\frac{p^*-r}{1-\beta}} (v\beta - r)\,dv, & if\ p^* > \frac{r}{\beta} \\ \int_{p^*}^{1} (v - p^*)\,dv, & otherwise \end{cases}$$

In the region where piracy exists, that is, when $p^* \ge \frac{r}{\beta}$, there are two terms in the above expression, the first term representing the surplus of legal users and the second, that of pirates. Substituting p^* from (2), we obtain

Proposition 3. (Consumer Surplus) *The consumer surplus, CS, is given by*

$$CS = \begin{cases} \frac{1+15\beta-30r}{32} + \frac{r^2}{32}\left(\frac{1}{1-\beta} + \frac{16}{\beta}\right), & if\ r < \rho_1 \\ \frac{(\beta-r)^2}{2\beta^2}, & if\ \rho_1 \le r < \rho_2 \\ CS_0 = \frac{1}{32}, & otherwise \end{cases}$$

In order to see how the price and consumer surplus change with r, we plot them in Figure 3. In panel (a) of this figure, we plot p^* and compare it with (1) *benchmark price* p_0, and (2) $\bar{p}_0 = \frac{1}{2}$, the pure *monopoly price* that would have been charged if the manufacturer sold directly to consumers. We find that, as long as the enforcement level is not very high—specifically, if $r < \rho_5 = \frac{3\beta}{4}$—the retail price in the presence of piracy or its threat is lower than p_0. In fact, as long as $\beta > \frac{1}{3}$, if r decreases beyond $\frac{\beta}{2}$, this price drops even below the monopoly price of $\bar{p}_0$. This seems quite intuitive. After all, in the presence of piracy or its threat, the legal channel is weakened by the competition from its own shadow. What is surprising is that there is actually a region $\rho_5 < r < \rho_2$ where p^* is larger than p_0 despite the threat of piracy (see Figure 3(a)). The dynamics of the manufacturer–retailer relationship are quite interesting in this region. The manufacturer, being the first mover, squeezes the retailer by forcing it to hold p^* fixed at the limit price of $\frac{r}{\beta}$. The manufacturer relents only after the demand becomes heavily depressed as a result of the rapidly rising p^*, to an extent that it now starts taking its toll not just on the retailer but also on the manufacturer itself.

In Figure 3(b), we plot the consumer surplus; for completeness, we also show the one excluding the illegal surplus as a dashed line. As expected, consumers are better off in the presence of piracy or its threat as long as the enforcement level is not too high, that is, $r < \rho_5$. What is interesting here is that, even though the retail price and total consumer surplus are monotonic in r in the piracy and threat regions (see Figure 3(b)), the consumer surplus excluding pirates is not necessarily so. More importantly, ρ_4 in Theorem 1 is always less than ρ_5, implying that the region where the manufacturer or retailer prefers piracy, consumers do too. This leads us to our next result.

Theorem 2. (Invisible Hand) *Let ρ_3 and ρ_4 be as above. Then, the manufacturer, retailer, and consumers are all better off in the presence of piracy or its threat, provided $\rho_3 < r < \rho_4$.*

We conclude this section by recalling that, in calculating the consumer surplus in Proposition 3, we included the surplus generated through piracy and illegal consumption. Even if we were to exclude this surplus, our result as stated in Theorem 2 would still hold (see Figure 3(b)). Either way—with or without the illegal surplus—our results so far, taken together, seem to suggest that, indeed, there exists an *invisible hand* of piracy! Even when every player is acting in his or her own narrow self-interest—the manufacturer and retailer maximizing their profits, and consumers their own utility—somehow, the presence of piracy or its threat is making every selfish actor better off. This invisible hand surely begs for an economic explanation, which we present next.

The Invisible Hand

Before we can understand how piracy lends an invisible hand, we need to recognize that the inefficiencies that exist in this supply chain are fundamentally rooted in the well-known issue of *double marginalization* or *vertical externality* (Tirole

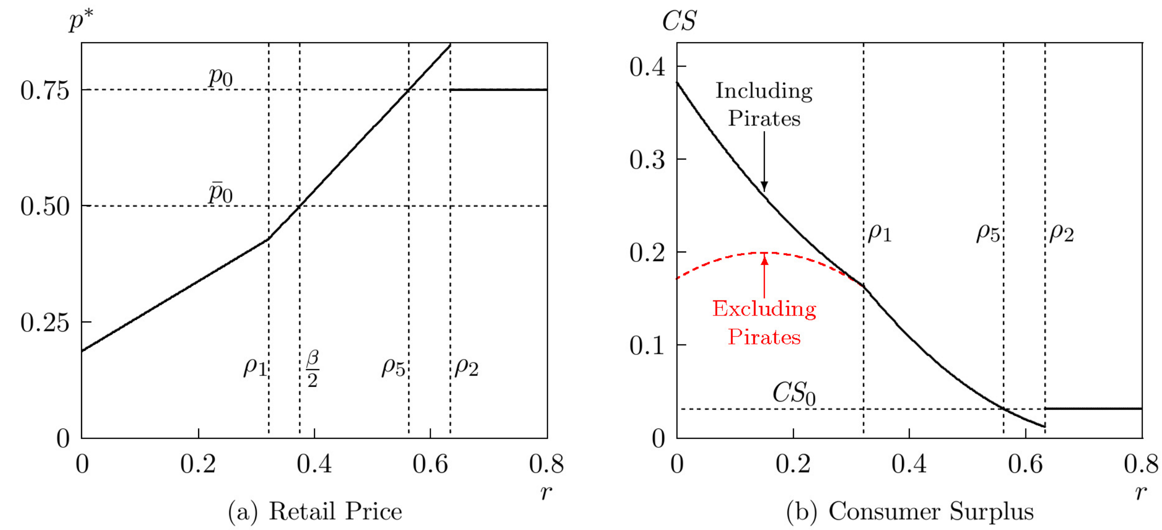

Figure 3. Equilibrium Retail Price and Consumer Surplus: $\beta = 0.75$

1992, p. 175). At the same time, information goods also suffer from piracy. What we find fascinating is that piracy or its threat can, to a degree, diminish the undesirable impact of double marginalization. At the core of this notion is the fact that double marginalization actually stems from the monopolistic nature of the market and a lack of competition at every stage of the supply chain. In contrast, when the market is competitive, its invisible hand would ensure that such inefficiencies are mitigated, resulting in socially desirable outcomes. What piracy provides is really a close proxy: it introduces a shadow competition for the manufacturer and the retailer, providing an alternative, a closely related version of the good, to the consumer.

Is piracy then equivalent to competition, and is it just another tool to combat double marginalization? A closer look reveals that the underlying process through which piracy curbs double marginalization is quite different from how up- or downstream competition works. When manufacturers compete against one another, such competition predictably squeezes them by cutting their pricing power and, thus, always enhances the retailer's market position. The gain in channel efficiency is totally captured by the retailer and not shared with the manufacturers at all. Likewise, with downstream competition, the retailers suffer as the gain accrues to the manufacturer alone. In other words, there can never be a win–win situation with traditional competition. Piracy, on the other hand, creates a shadow competition for the entire legal channel, and not just for the manufacturer or the retailer: every time the channel loses a consumer to piracy, both the manufacturer and retailer suffer. The first order effect of this shadow competition is a simultaneous reduction in the pricing power of both parties. The second order effect is also simultaneous but, by nature's justice, works in the opposite direction: when the manufacturer responds to the competition by lowering its markup, it inadvertently ends up helping the retailer too; likewise, when the retailer responds, the manufacturer also benefits. Interestingly, it turns out that, at moderate levels of enforcement—not too much, not too little—this second order effect can more than compensate for the first order effect, thereby increasing the profits for both of them and creating a win–win situation. At high and low levels of enforcement, however, the second order effect is not as pronounced, leading to lower profits for both. Indeed, as stated in Theorem 1, piracy, depending on its prevailing level, can result in a win–win, win–lose, or lose–lose situation.

Now, how should a central planner or a policymaker react in this situation? The invisible hand of a competitive market, after all, is supposed to bring about a socially desirable outcome (Smith 1776, p. 477). Does piracy have a similar impact? In order to answer this and obtain a more complete picture, let us now investigate the impacts on channel profit and social welfare.

Channel Profit and Social Welfare

The channel profit (*CP*) is the total profit generated by the manufacturer and retailer together, and social welfare (*SW*) for a zero marginal cost good is obtained by simply aggregating all its consumption benefits.

Proposition 4. (Channel Profit and Social Welfare) *Let ρ_1 and ρ_2 be as* above. *In equilibrium, the channel profit (CP) and social welfare (SW) are respectively given by*

$$CP = \begin{cases} \frac{3(1-\beta+r)^2}{16(1-\beta)}, & if\ r < \rho_1 \\ \frac{r(\beta-r)}{\beta^2}, & if\ \rho_1 \le r < \rho_2 \\ CP_0 = \frac{3}{16}, & otherwise \end{cases}$$

and

$$SW = \begin{cases} \frac{7+9\beta+6r}{32} - \frac{r^2}{32}\left(\frac{1}{1-\beta} + \frac{16}{\beta}\right), & if\ r < \rho_1 \\ \frac{1}{2}\left(1 - \frac{r^2}{\beta^2}\right), & if\ \rho_1 \le r < \rho_2 \\ SW_0 = \frac{7}{32}, & otherwise \end{cases}$$

Comparing them with their benchmark values, we obtain the following important result:

Proposition 4. (Impacts on Channel Profit and Social Welfare) *Let $\rho_5 = \frac{3\beta}{4}$ be as defined earlier. Further, let*

$$\rho_6 = \begin{cases} \sqrt{1-\beta} - (1-\beta), & if\ \beta \le \frac{8}{9} \\ \frac{\beta}{4}, & otherwise \end{cases}$$

Then, in the presence of piracy or its threat, the channel profit is higher for $\rho_6 < r < \rho_5$, and the social welfare is higher for $0 \le r < \rho_5$, when compared to their respective benchmark values.

Theorem 3 is better visualized in Figure 4, where we plot the channel profit and social welfare as functions of the enforcement level. Together, they show that, over a significant portion of the parameters space, the supply chain and the entire society perform better in the presence of piracy or its threat than without, irrespective of whether the pirates' surplus is included in the analysis. Such results could give policymakers a reason for a momentary pause, perhaps to ponder whether to tolerate some piracy and exercise moderation when stepping up enforcement.

At the same time, we must also recognize that, although piracy injects a proxy competition into the market, it has its own obvious disadvantage. After all, it is an illegal activity and cannot be tolerated unabated. Moreover, when the enforcement level is too low and piracy is rampant, it eats deep into the surplus of the legal channel, which may be a cause for concern for a policymaker interested in the overall health of the industry. At the other extreme, though, when the enforcement level is very high, the diminished threat of piracy may not mitigate the problem of double marginalization at all; in fact, Figure 4 clearly shows that it exacerbates the problem when $\rho_5 < r < \rho_2$. It is only when the enforcement level is moderate, and the piracy rate low, that the shadow competition from piracy can truly mimic the invisible hand of a competitive market.

Channel Efficiency

How *efficient* is piracy in mitigating vertical externality? To address this issue in a rigorous manner, it is necessary that we provide an analysis of the vertically integrated setting, a supply chain where the manufacturer and retailer operate together as one entity. The profit-maximization problem faced by the integrated firm, conveniently referred to as the manufacturer henceforth, is $\max_{\bar{p}} \bar{\pi}(\bar{p}) = \bar{p}q(\bar{p})$. Similar to what we have seen in the previous section, the equilibrium outcomes can again be characterized in terms of three separate regions.

Proposition 5. (Equilibrium for Integrated Firm) *Let $\bar{\rho}_1 = \frac{\beta(1-\beta)}{2-\beta}$ and $\bar{\rho}_2 = \frac{\beta}{2}$. Then, $\bar{\rho}_1 < \bar{\rho}_2$, and the following three cases emerge:*

- **Piracy Region**: *When $r < \bar{\rho}_1$, the manufacturer finds it optimal to tolerate some level of piracy. In this case, $\bar{p}^* = \frac{1-\beta+r}{2}$ and $\bar{\pi}^* = \frac{(1-\beta+r)^2}{4(1-\beta)}$.*
- **Threat Region**: *When $\bar{\rho}_1 \le r < \bar{\rho}_2$, there is no piracy, but the threat of piracy affects the pricing decision of the manufacturer. Here, $\bar{p}^* = \frac{r}{\beta}$ and $\bar{\pi}^* = \frac{r(\beta-r)}{\beta^2}$.*
- **Benchmark Region**: *For $r \ge \bar{\rho}_2$, even the threat of piracy disappears, resulting in $\bar{p}^* = \frac{1}{2}$ and $\bar{\pi}^* = \frac{1}{4}$.*

The story that emerges from a comparison of the channel profits in Propositions 4 and 5 is quite interesting. In Figure 5(a), when the legal channel is vertically integrated—and the issue of double marginalization absent—the channel profit increases in the level of enforcement until the point where the issue of piracy completely disappears. In other words, $\bar{\pi}^*$ increases in *r*, just as prior research suggests. Also, beyond the $\bar{\rho}_2$ boundary, $\bar{\pi}^*$ stays unchanged at $\frac{1}{4}$, and this profit is the ideal outcome for the legal channel, as neither double marginalization nor piracy is present any longer. In contrast, for the manufacturer–retailer chain, the channel profit is not monotonic in *r*. The channel profit, $(\pi_m^* + \pi_r^*)$, initially in-

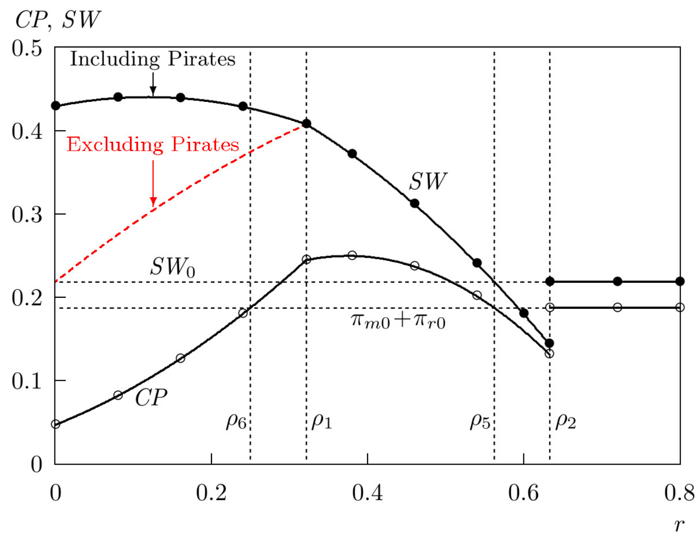

Figure 4. Channel Profit and Social Welfare: $\beta = 0.75$

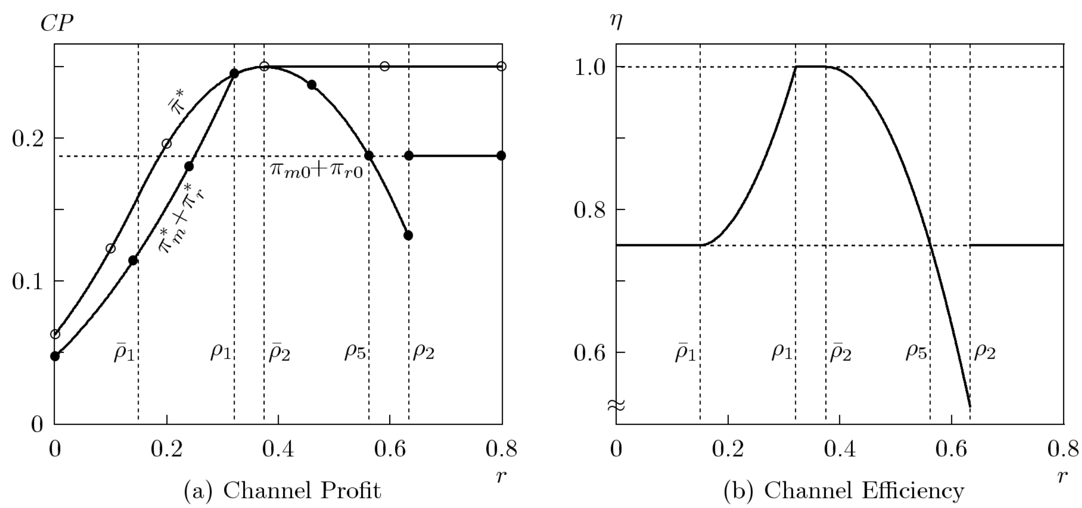

Figure 5. Manufacturer–Retailer Chain Versus Vertically Integrated Firm: β = 0.75

creases with r, just as the conventional wisdom suggests. This is because the pirated good becomes less attractive to consumers, allowing the legal channel to partly reclaim its pricing power. Beyond a certain threshold of r, however, the pricing power has been reclaimed to an extent that the problem of double marginalization starts to dominate; now, $(\pi^*_m + \pi^*_r)$ decreases in r. Eventually, when r approaches the ρ_2 boundary, the channel profit plunges because of severe double marginalization. The net result, in short, is the lack of monotonicity in the profit plot to the left of ρ_2. Finally, when $r > \rho_2$, the issue of piracy disappears, but double marginalization remains, presenting itself as the gap between $(\pi_{m0} + \pi_{r0})$ and $\frac{1}{4}$ in Figure 5(a).

In order to see the extent to which piracy mitigates double marginalization, we now compare the profits of the decentralized and integrated channels. This comparison is summarized in Figure 5(b) using the notion of channel efficiency, η, which is essentially the ratio of the two, that is, $\eta = \frac{(\pi^*_m+\pi^*_r)}{\bar{\pi}^*}$.

Theorem 4. (Channel Efficiency) *Let* $\overline{\rho}_1$, ρ_1, $\overline{\rho}_2$, *and* ρ_5 *be as above. Then, piracy or its threat improves the channel efficiency by suppressing the impact of double marginalization as long as* $\overline{\rho}_1 < r < \rho_5$. *In fact, for* $\rho_1 \leq r \leq \overline{\rho}_2$, *the channel efficiency reaches 100%.*

Theorem 4 allows us to further narrow down the region where piracy is most efficient in coordinating the supply chain. The bound placed on r in Theorem 4 simply suggests that the enforcement level must be moderate: it cannot be too low or too high. It must also be noted that the interval $[\rho_1, \overline{\rho}_2]$ is non-empty only if $\beta \geq \frac{2}{3}$. This lower bound on β simply means that, for the pirated version to be able to completely eliminate the effect of double marginalization, quality wise, it must be somewhat competitive against the legal product.

Endogenizing the Enforcement Level

What we find so far is that, unlike prior literature, piracy in our setting can positively impact both private profits and public welfare at the same time. Given the tension between private profits and public welfare recognized in prior research (e.g., Chen and Png 2003), it is natural to ask if this alignment of incentives is actually perfect. Put differently, would different parties actually seek the same level of enforcement in a given context and, if not, how would they differ?

Figure 6 summarizes all of our earlier results and shows in one place all of the winning windows for the different parties in our setup: compared to the benchmark region, the manufacturer prefers an r in (ρ_3, ρ_2), the retailer an r in (ρ_3, ρ_4), and the channel as a whole, an r in (ρ_6, ρ_5). The consumer and social surplus, on the other hand, are both higher than their respective benchmark values for $r \in [0, \rho_5)$. It is easy to show that, out of the five windows shown in Figure 6, four of them—manufacturer's, channel's, consumers', and social planner's—always exist regardless of the value of β. Only the retailer's window vanishes for a very high β—specifically, for $\beta \geq \frac{8}{9}$ in our setup—implying that this window, too, is guaranteed to exist for $\beta < \frac{8}{9}$.

Endogenizing r, we can also find the optimal level of enforcement for each of the parties above:

Lemma 2. *The manufacturer would like to set enforcement at* $r_m^* = \frac{\beta(3-2\beta)}{2(2-\beta)}$, *the channel at* $r_c^* = \frac{\beta}{2}$, *consumers at* $r_C^* = 0$, *and the policymaker at* $r_S^* = \frac{3\beta(1-\beta)}{16-15\beta}$. *The retailer would prefer* $r_r^* = \frac{3\beta(1-\beta)}{4-3\beta}$ *for* $\beta < \frac{8}{9}$.

Figure 6 shows these optimal points as red dots. With simple algebra, the observed order of these points in the figure, $r_C^* < r_S^* < r_r^* < r_c^* < r_m^*$, can actually be guaranteed for all $\beta \in (0, 1)$. Now, because $r_r^* < r_c^* < r_m^*$, the divergence within the channel itself is clear: the manufacturer prefers more enforcement than the retailer and the channel. Moreover, because $r_S^* < r_r^*$, there is a divergence of incentives beyond the channel as well. Proposition 6 states this formally.

Proposition 6 (Private Profit Versus Public Welfare) *The level of enforcement which maximizes the consumer or social surplus is always less than the level at which the profit of the manufacturer, the retailer, or the overall channel is maximized.*

Before concluding this section, we wish to make a quick note. Lemma 2 and Proposition 6 ignore the cost of piracy enforcement. However, in reality, governments do incur significant costs in implementing various antipiracy laws, implying that policymakers might actually prefer an even lower level of enforcement. Consequently, in practice, a serious gap might exist between what the manufacturer or retailer wants and how governments actually respond.

Heterogeneity in Piracy Cost

So far, we have assumed that all consumers have exactly the same piracy cost, r. How would our results change if consumers incur different costs for pirating? Indeed, on top of their heterogeneity in v, consumers may also be heterogeneous in their piracy cost. Consumers with differing levels of technical skills may incur different acquisition costs; likewise, different consumers may have different expectations about the legal penalty. To capture this, we adopt a discrete model of heterogeneity. We assume that consumers are of two types, *high* and *low*, with probability α and $(1 - \alpha)$, respectively. Further, the low type faces a piracy cost of $r_L = r$ and high, of $r_H = g + hr$, where $g \geq 0$ and $h \geq 1$. Essentially, g represents an additional piracy cost that is intrinsic to the high type regardless of the level of enforcement, whereas hr represents the piracy cost that depends on enforcement.

It is easy to see that our original IR and IC constraints will continue to apply for the low type. However, for the high type, only (IR-Legal) will hold now; the others will change to

$$v - p \geq v\beta - (g + hr) \Rightarrow v \geq \frac{p-(g+hr)}{1-\beta} \qquad \text{(IC-Legal-High)}$$

$$v\beta - (g + hr) \geq 0 \Rightarrow v \geq \frac{(g+hr)}{\beta} \qquad \text{(IR-Pirated-High)}$$

$$v\beta - (g + hr) > v - p \Rightarrow v < \frac{p-(g+hr)}{1-\beta} \qquad \text{(IC-Pirated-High)}$$

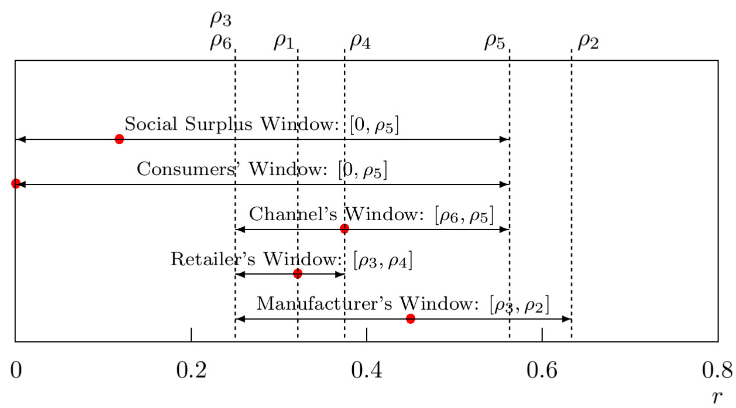

Figure 6. Different Winning Windows: β = 0.75

With the inclusion of high-type consumers into the mix, one additional possibility arises—p may now be set so high that $\frac{p-r}{1-\beta} \geq 1$; in that case, no one of the low type buys the legal product, and the legal demand comes exclusively from the high segment. Such a scenario is indeed expected when α is large and the legal channel has little incentive to also cater to the low segment. Recognizing this, we can revise the demand expression in (1) as follows:

$$q(p)=\begin{cases} 1-p, & \textit{Case R0}: p<\frac{r}{\beta}<\frac{g+hr}{\beta} \ (\textit{Benchmark}) \\ \alpha(1-p)+(1-\alpha)\left(1-\frac{p-r}{1-\beta}\right), & \textit{Case R1}: \frac{r}{\beta}\leq p<\frac{g+hr}{\beta} \ \textit{and}\ \frac{p-r}{1-\beta}\leq 1 \\ \alpha(1-p), & \textit{Case R2}: \frac{r}{\beta}\leq p<\frac{g+hr}{\beta} \ \textit{and}\ \frac{p-r}{1-\beta}>1 \\ \alpha\left(1-\frac{p-(g+hr)}{1-\beta}\right)+(1-\alpha)\left(1-\frac{p-r}{1-\beta}\right), & \textit{Case R3}: p\geq\frac{g+hr}{\beta} \ \textit{and}\ \frac{p-r}{1-\beta}\leq 1 \\ \alpha\left(1-\frac{p-(g+hr)}{1-\beta}\right), & \textit{Case R4}: p\geq\frac{g+hr}{\beta} \ \textit{and}\ \frac{p-r}{1-\beta}>1 \end{cases} \quad (3)$$

Clearly, the retailer's problem becomes a bit more complex with this new demand function in (3), as it has to consider these five alternatives and choose the one that maximizes $(p - w)q(p)$. The manufacturer, as before, would anticipate the retailer's reaction and set the wholesale price accordingly. The manufacturer's choice would eventually compel the retailer to choose one of the five strategies above, making all five a possibility in equilibrium. Further, six additional possibilities with corner solutions must be accounted for. In those limiting situations, the manufacturer can actually compel the retailer to choose one of the following limiting values of p:

- When in Case R1, the retailer may be forced to set $p=\frac{r}{\beta}$; this case is named R1A.
- When in Cases R1 and R3, the retailer may be compelled to choose $p = 1 - \beta + r$, ensuring that $\frac{p-r}{1-\beta}$ is exactly equal to 1. We call these cases R1B and R3A, respectively.
- When in R3 and R4, the manufacturer can force the retailer to set $p=\frac{g+hr}{\beta}$, making all high-type consumers refrain from piracy. These cases are labeled R3B and R4A, respectively.
- When in R4A, if the interior w exceeds p, then the manufacturer would be compelled to charge $w=\frac{g+hr}{\beta}$, resulting in Case R4B.

In all, we end up with a total of 11 possible cases in equilibrium, 5 of them with interior solutions and 6 with corner solutions; for details, see Appendix A. This extended model, although significantly more complicated than before, still remains analytically tractable and, fortunately, closed-form solutions for all the 11 equilibrium cases can be found after a careful analysis. However, a thorough description of the analysis turns out to be quite tedious and, in fact, somewhat unnecessary for our purpose. It suffices to simply illustrate the relevant partitions of the parameter space in terms of the 11 equilibrium outcomes; Figure 7 depicts these partitions for "moderate" values of g and h.

After comparing this solution to our original one, it becomes clear that R1A is equivalent to the original *threat region*,

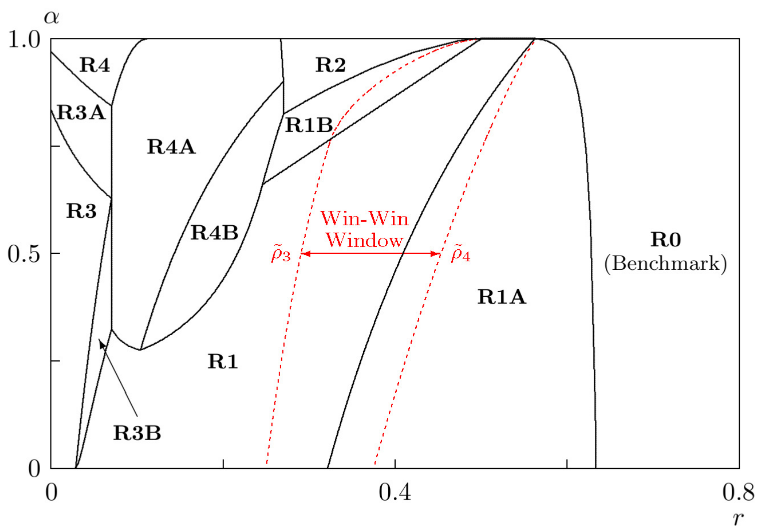

Figure 7. Partitions of the (r, α) Space: $g = 0.1$, $h = 2$, $\beta = 0.75$

where piracy is barely eliminated but the threat of piracy stops the legal channel from pricing the product too high. In all regions except R0 and R1A, piracy exists. Only the low type pirates in R1, R1B, R2, R4A, and R4B, whereas both segments engage in piracy in R3, R3A, R3B, and R4. Further, in six of the regions—R1B, R2, R3A, R4, R4A, and R4B—the retail price is set so high that low-type consumers do not buy the legal product and only resort to piracy.

Fortunately, our primary insight that there is a surprising win–win region that spans parts of the piracy and threat regions remains valid across all $\alpha < 1$, and a region analogous to (ρ_3, ρ_4) in Theorems 1 and 2 can be found (see Figure 7). This new region is denoted $(\tilde{\rho}_3, \tilde{\rho}_4)$,where

$$\tilde{\rho}_3 = \begin{cases} \frac{\sqrt{(1-\beta)(1-\alpha\beta)}-(1-\beta)}{1-\alpha}, & \text{if } \alpha \le \frac{7-8\beta+\sqrt{49-48\beta}}{32\beta(1-\beta)} \\ \frac{\sqrt{2\alpha(1-\alpha)(1-\alpha\beta)(2\alpha\beta-1)}}{4\alpha(\alpha(2\beta-1)-1)} + \frac{1-2\beta-3\alpha\beta+4\alpha\beta^2}{2(\alpha(2\beta-1)-1)}, & \text{otherwise} \end{cases}$$

and

$$\tilde{\rho}_4 = \beta\left(1-\tfrac{1}{4}\sqrt{\tfrac{1-\alpha\beta}{1-\beta}}\right)$$

Interestingly, both $\tilde{\rho}_3$ and $\tilde{\rho}_4$ are independent of g and h; however, they are valid only if both g and h are not very small. Of course, when they are both small, the two consumer types—high and low—become indistinguishable, and we get back the results of our original analysis. When both g and h are not small, $\tilde{\rho}_3 < \tilde{\rho}_4$ holds (as long as β is not too large), indicating that the win–win window exists irrespective of the value of α. Further, for all $\alpha \le \frac{7-8\beta+\sqrt{49-48\beta}}{32\beta(1-\beta)}$, the win–win window is increasing in α, that is, $\frac{\partial(\tilde{\rho}_4-\tilde{\rho}_3)}{\partial\alpha} > 0$, implying that this window can actually widen in the presence of heterogeneity in the piracy cost. This is also seen in Figure 7, where the threshold curves clearly move apart as α increases within a limit.

We end this section by noting that, in certain situations, there may be some purely "ethical" consumers who stay out of piracy under all circumstances (August and Tunca 2008; Lahiri and Dey 2013). Such a situation is actually a special case of this extension. Specifically, if $g \ge \beta$, no consumer of the high type would be able to satisfy (IR-Pirated-High); these consumers would, therefore, be forced to remain ethical. In that case, regions R3, R3A, R3B, R4, R4A, and R4B would completely disappear. The win–win window of ($\tilde{\rho}_3, \tilde{\rho}_4$) persists, however.

Other Considerations

In this paper, we deliberately set up a simple model to focus on the interaction between piracy and double marginalization. In so doing, we ignored several issues that may also be important in an information-good supply chain. We now consider some of them to further verify generalizability of our results and seek newer insights.

Commercial Pirates

We first consider the case of commercial pirates. Prior literature has documented the existence of such pirate suppliers who strategically set a price for the pirated version (Tunca and Wu 2013). Let this price be $s > 0$, so that the effective price to a consumer for the pirated version is now $(r + s)$. The sequence of events is similar: the manufacturer sets w; the retailer then names p, followed by the pirate supplier setting s; finally, consumers decide whether to buy, pirate, or forgo use.

When this game is solved, we find that there exists $\tilde{\rho}_2, \tilde{\rho}_3$, and $\tilde{\rho}_4$ such that, for $r < \tilde{\rho}_2$, three regions emerge: (1) a lose–lose region for $r \in [0, \tilde{\rho}_3]$, where the manufacturer and retailer are both better off without piracy, (2) a win–lose region for $r \in [\tilde{\rho}_4, \tilde{\rho}_2)$, where only the manufacturer is better off with piracy but the retailer is not, and most importantly, (3) a win–win region for $r \in (\tilde{\rho}_3, \tilde{\rho}_4)$, where the manufacturer and retailer are both better off with piracy than without. These results are better visualized in Figure 8 and can be readily compared to the original ones in Figure 2.

There are two points worth noting in these plots. First, when the pirate supplier is strategic, the win–win region not only persists but also expands—in Figure 8(b), the area between $\tilde{\rho}_3$ and $\tilde{\rho}_4$, is larger than the area between ρ_3 and ρ_4 in Figure 2. Second, the limit for β beyond which the win–win region disappears also becomes larger in the presence of a commercial pirate. In summary, our results only get stronger with commercial pirates. The detailed analysis of this extension is provided in the online supplement to this paper.

Subscription Services and Product Bundling

Our basic model was set up for a single information good. Naturally, one may ask how our model extends to subscription services with multiple goods. For supply chains that involve a single manufacturer and a single retailer—for example, for a chain involving HBO and Comcast—the original setup extends naturally by interpreting p and w not as one-time charges but as charges per unit time. Under this interpretation, HBO charges Comcast a monthly fee of w for its collective offering on the channel, the entire collection becoming a single product in effect. Comcast then adds its markup to set a monthly subscription fee of p that it charges its consumers for the HBO channel. Given the existence of double marginalization in this setup (Caves 2005, p. 235) and the abundance of pirated HBO content, clearly, our basic model can provide useful insights into such a situation.

Now, what would happen if the retailer bundles multiple goods from multiple manufacturers? This conceptualization is certainly appropriate for subscription services such as Netflix that combine content from multiple providers into a single subscription service. In order to rigorously examine this possibility, we now consider a retailer who combines content from two manufacturers, 1 and 2, at a monthly fee of w_1 and w_2 respectively, and charges consumers a monthly subscription rate of p for the content bundle.[8]

Assuming that the consumers' valuation for the bundle still follows a uniform distribution over [0, 1], and the same degradation factor, β, for both types of pirated content, it can be verified that a win–win window similar to the one in Theorem 1 exists. This window is given by ($\tilde{\rho}_3, \tilde{\rho}_4$) in Figure 9. In other words, we find that the win–win window exists even when the retailer bundles two products from two manufacturers. In fact, comparing Figure 9 with Figure 2, we can easily see that the win–win window actually expands when the retailer bundles. The threshold for β beyond which the win–win window disappears also increases. For details, see Appendix B.

Piracy Cost Recouped by the Legal Channel

The expected piracy cost incurred by all the pirates collectively can be expressed as $r\overline{q}(p)$, where $\overline{q}(p)$ is the illegal demand given in (1). In our basic setup, we assumed that this penalty cannot be recouped, not even partially, by the legal channel. However, in reality, it is entirely possible that at least a portion of the penalty is recovered by the legal channel and is shared by the manufacturer and the retailer in some way. To see if our results hold under such a setting, we now allow a $\lambda \in [0. 1]$ fraction of the piracy cost, that is, $\lambda r\overline{q}(p)$, to be transferred to the legal channel; the manufacturer gets a μ fraction of that transfer, while the remaining $(1 - \mu)$ fraction is claimed by the retailer.

Finding the equilibrium solution for this sequential game remains conceptually straightforward. Although the algebra is a bit more involved, it is still possible to obtain closed-form

[8]To be sure, we do not endogenize the bundling decision itself. The reason is simple. If the retailer decides not to bundle, our earlier result can be applied directly to each product (or channel) separately. If, on the other hand, the retailer does bundle, the analysis in this section would soon show that, qualitatively, our earlier results continue to hold. In other words, irrespective of whether the retailer chooses to bundle, our results are robust, making the bundling decision itself inconsequential in our setup. Also, we do not consider bundling by the manufacturer; if the manufacturer bundles multiple products or channels, the entire bundle can be viewed as a single product, and we are then back to our original setup.

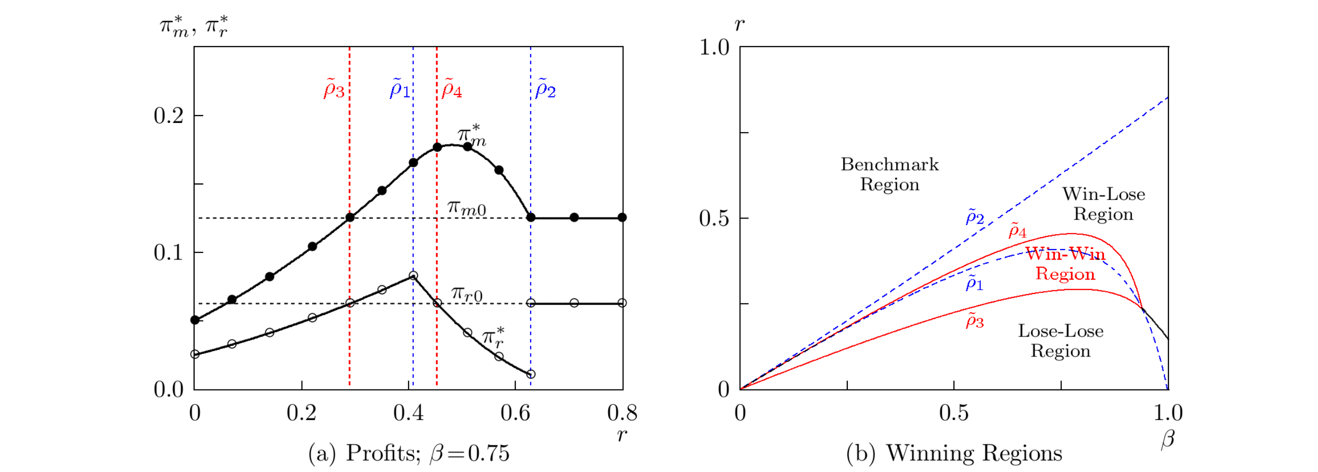

Figure 8. Manufacturer's and Retailer's Profits and Winning Regions with Commercial Pirates

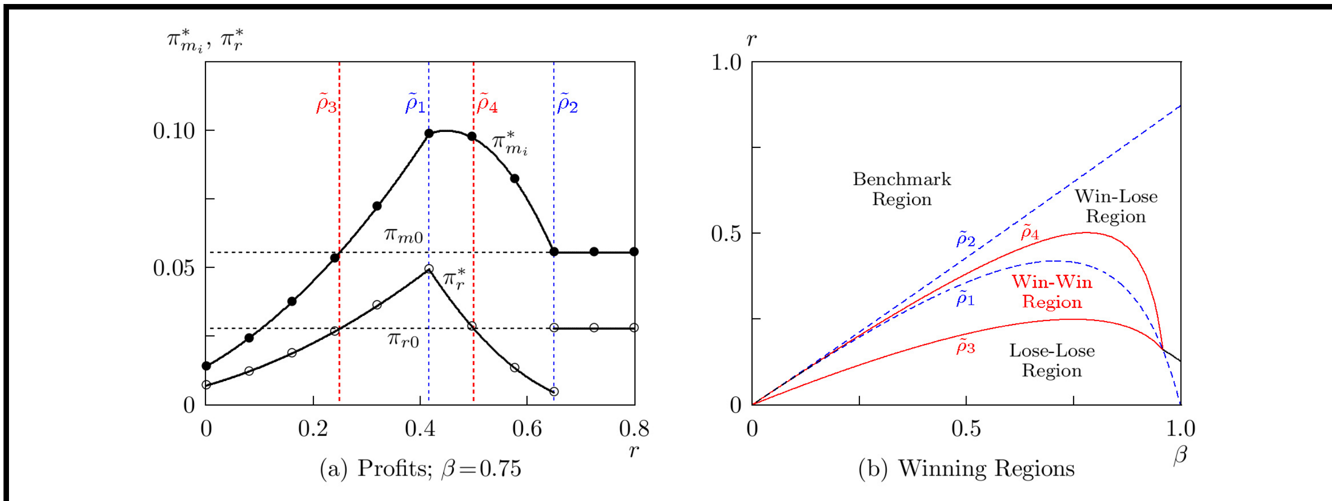

Figure 9. Manufacturer's and Retailer's Profits and Winning Regions with Bundling

solutions (see Appendix B). It turns out that, except for extreme values of λ and μ, our results in Theorems 1 and 2 largely hold in this extended setup, and a win–win situation does exist for a large part of the parameter space. Specifically, it is possible to find closed-form solutions for thresholds $\tilde{\rho}_3$ and $\tilde{\rho}_4$ —analogous to ρ_3 and ρ_4 in Theorems 1 and 2—and, as long as (1) $\tilde{\rho}_3$ is real, (2) $\tilde{\rho}_4$ is real, and (3) $\tilde{\rho}_3 < \tilde{\rho}_4$, a win–win region of ($\tilde{\rho}_3, \tilde{\rho}_4$) is guaranteed to exist for the legal channel. For the win–win region to disappear, at least one of these three conditions must be violated. This is illustrated in Figure 10, where the entire (λ, μ) space has been partitioned into two main zones: one where all three conditions are met and the win–win region exists, and another consisting of three subregions (separated by dashed lines when necessary), each violating one of the conditions. The region in the bottom-right corner of the plot violates the condition that $\tilde{\rho}_4$ is real, and those at the top-right violate one of the remaining two conditions. The end result is that, for a large part of the parameter space, a win–win region is manifest, making our original results robust to this extension. Specifically, when λ is not too large, that is, when the transfer of penalty to the legal channel is not substantial, our earlier results continue to hold for all values of μ. However, as λ becomes sufficiently large, our results hold only for a narrower range of μ. In fact, if λ is large, at extreme values of μ, either the manufacturer wins or the retailer does, but the region where they both do shrinks.

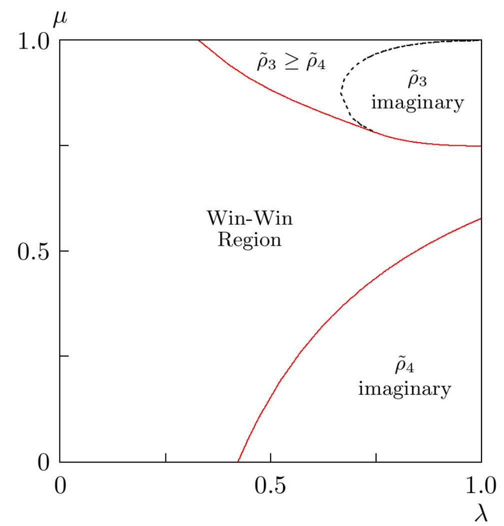

Figure 10. Partitions of the (λ, μ) Space: $\beta = 0.75$

We end this section with an interesting observation. As a special case of this extension, we can imagine a situation where the legal channel uses versioning to eliminate piracy altogether. Indeed, this can be accomplished by selling a degraded version of quality β at a retail price of r.[9] Clearly, when versioning is introduced in this market, the demand for the pirated copy would drop to zero, as all pirates would now switch to the lower-quality legal version, without further cannibalizing the demand for the higher one. Therefore, the demands for the higher- and lower-quality versions would respectively become $q(p)$ and $\overline{q}(p)$, exactly as defined in (1). Now, assuming that the manufacturer charges a wholesale price of s for the lower version, it can be verified that this is actually a special case where $\lambda = 1$ and $\mu = \frac{s}{r}$.

Network Effect

Many information goods exhibit a positive network effect that enhances a consumer's valuation for the product. We now examine whether our results remain applicable in the presence of such a network effect. There is another point behind this exercise. Prior research has shown that a positive network effect can induce the manufacturer to tolerate more piracy (Conner and Rumelt 1991); we would like to see if this effect extends to the retailer as well.

[9]The retail price for the lower version must be exactly r: there would be no market for the lower version if the retail price were any higher; and if it were any lower, the legal channel would have left money on the table.

Specifically, we now assume that a consumer's valuation is $v(1 + \Gamma)$, where $\Gamma > 0$ is the expected network effect. We consider this network effect to be proportional to the total number of consumers, legal and pirated combined, and express it as $\Gamma = \gamma(1 - \overline{v})$, where $\overline{v}$ is the marginal user indifferent between using and not using the product and $\gamma > 0$ represents the strength of the network effect.

We omit the tedious algebra for brevity. To illustrate the impact of γ, we plot in Figure 11 the $\tilde{\rho}_i$ thresholds, $i \in \{1, 2, 3, 4\}$, on the (r, γ) space. As before, when r is below $\tilde{\rho}_1$, we have the piracy region; between $\tilde{\rho}_1$ and $\tilde{\rho}_2$, we have the threat region, and above $\tilde{\rho}_2$, the benchmark region. It is evident from Figure 11 that $\tilde{\rho}_1$ increases with γ. This is expected as the network effect provides the legal channel a greater incentive to tolerate piracy. Likewise, $\tilde{\rho}_2$ increases as well, implying that the benchmark region also shrinks predictably. Thus, our results are in conformity with prior literature (Conner and Rumelt 1991).

Of particular interest is the interval ($\tilde{\rho}_3, \tilde{\rho}_4$), where both the manufacturer and retailer make at least as much in profits as they do in the benchmark region. It is clear from Figure 11 that this region expands with γ, which simply means that the presence of a network effect elevates the relative appeal of piracy. This is also not surprising since piracy, in addition to arresting double-marginalization, now provides an additional network benefit: illegal usage enhances the market coverage, leading to a higher valuation, a portion of which the legal

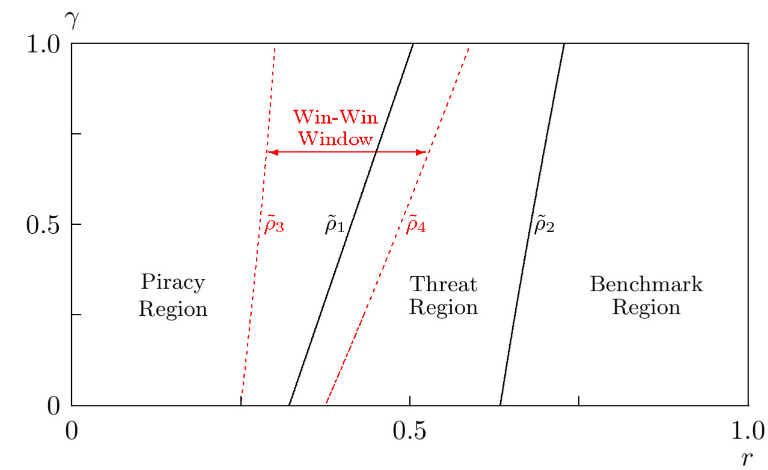

Figure 11. Partitions of the (r, γ) Space: $\beta = 0.75$

channel then extracts through higher prices. Overall, this extended model shows that our earlier results are indeed robust and that their relevance simply increases when a positive network effect is present. Technical details are found in the online supplement.

Downstream Competition

We now consider the issue of downstream competition, that is, competition among retailers. In keeping with the realities of these markets (see, for example, Table 1), we treat this competition as imperfect and allow our retailers to retain some level of pricing power. This imperfection, as mentioned in the introduction, could originate from a variety of sources. Brand names, brick-and-mortar presence, recommender systems, loyalty programs and other lock-in mechanisms, and long-tails of product portfolios may all play a role. Imperfection may also arise in a cross-channel competition where two retailers are reselling the same good but in different formats; for example, one retailer could carry a movie in the Blu-ray format, whereas another could sell it as a download. Since consumers have different preferences for different formats, they may like one retailer more than the other.

Capturing each of these sources of imperfection in a single model is not practical. Rather, our purpose is better served by a simpler model that can capture the essence of imperfection through one single parameter, allowing us to study the interplay between piracy and double marginalization at varying levels of competition. To this end, we consider a model with two retailers, A and B, located at the two extremities of a linear market of unit length (maximal differentiation); they charge a price of p_A and p_B, respectively, for the same product. Consumers are assumed to be uniformly distributed along this linear market and are indexed by their location $x \in [0, 1]$. To capture consumer x's preference toward one retailer over the other, we express his fit cost as $x\delta$ and $(1 - x)\delta$ for retailers A and B, respectively. Therefore, the fit cost per unit length, $\delta > 0$, captures the extent of imperfection; by varying δ, we can study the entire spectrum—from perfect competition, $\delta \to 0$, all the way to pure monopoly, $\delta \to \infty$.

Although a bit more complicated than before, this game can also be solved. By comparing the manufacturer's and retailers' profits with their benchmark values, we can find their winning windows, $(\tilde{\rho}_{3m}, \tilde{\rho}_2)$ and $(\tilde{\rho}_{3r}, \tilde{\rho}_4)$, respectively. When these two windows are plotted together in Figure 12, it becomes plainly visible that there is an overlap between the two windows for a fairly large portion of the parameter space, implying that the win–win window is still possible. As long as the competition is not too fierce, that is, as long as δ is not too small, the win–win window exists, where the manufacturer and both retailers are better off in the presence of piracy or its threat than without. Furthermore, when piracy puts a downward pressure on the retail price, it can never hurt consumer welfare, so consumers too are better off in this region. In summary, our earlier findings in Theorems 1 and 2—a moderate level of piracy can make all parties better off— continues to hold in this setting.

It is reassuring to find that the win–win window converges to (ρ_3, ρ_4) when δ is large, that is, when the retailers are local monopolies. Now, as δ starts decreasing and the competition heats up, the impact of double marginalization lessens a bit. Predictably, the need for piracy to mitigate the vertical externality reduces, resulting in a shrinkage of the window where

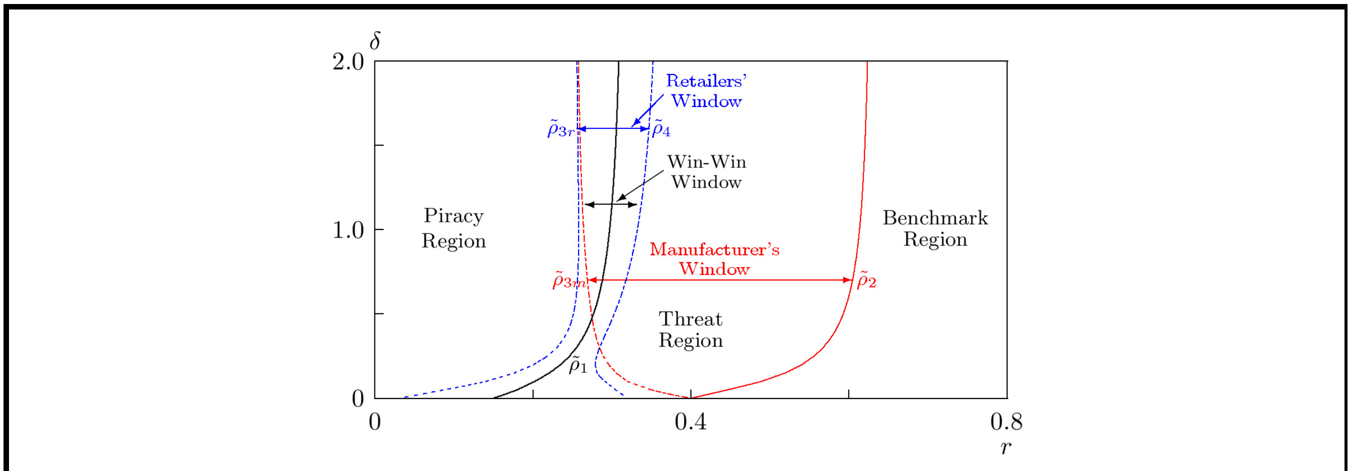

Figure 12. Partitions of the (r, δ) Space: $\beta = 0.75$

all parties can be better off, that is, where all are willing to tolerate piracy to a certain level. When δ decreases even further, at some point, the competition among retailers becomes too fierce, and double marginalization too weak to seriously hurt the manufacturer. It is only then that the win–win window disappears completely.

Interestingly, even though the overlapping window disappears at high levels of competition, the individual winning windows actually persist. Irrespective of the value of δ, that is, irrespective of the level of competition, the manufacturer and both retailers retain their own winning windows; it is simply that, at low values of δ, their windows move away from each other. Essentially, a decreasing δ starts causing a divergence of incentives between the manufacturer and retailers, pulling their winning windows apart. When δ is small, a fiercer downstream competition reduces the retailers' margins and makes the manufacturer better off even without piracy. Thus, as the pressure from double marginalization eases, the manufacturer becomes less tolerant toward piracy and starts seeking more enforcement in the form of a higher r; this causes $\tilde{\rho}_{3m}$, the left threshold of the manufacturer's window, to shift to the right, vis-à-vis the retailers' right threshold, $\tilde{\rho}_4$. Squeezed by fiercer competition, the retailers, on the other hand, find that a higher dose of piracy—or a relatively lower r—is needed to limit the manufacturer's market power to their advantage; this causes the retailers' window to expand. It is this divergence of incentives that manifests itself in the form of a shrinking win–win window as δ decreases. Therefore, consistent with our original setup, piracy still has a positive impact on the profits of every party, although a divergence in their incentives forces their preferred regions apart, shrinking the win–win window and eventually making it disappear. This insight that an increasing level of competition leads to divergent attitudes toward piracy for the manufacturer and retailers is also new and adds to the extant literature on this topic.

Finally, we revisit the difference between how piracy and downstream competition impact double marginalization. We already discussed this subtle, but key, difference, with respect to perfect competition. This difference becomes even more conspicuous when we consider imperfect competition, since the parameter δ gives us a lever by which we can vary the level of competition. As δ decreases, competition heats up and the resulting channel profit either increases or remains the same. In other words, more downstream competition is always better for the channel. The case of piracy is starkly different, however. As piracy increases, that is, as r decreases, the channel profit may increase in a certain range but decreases beyond. In other words, the impact of piracy is not uniform. Therefore, if the objective is simply channel coordination, an increase in the level of downstream competition is always desirable, but only a moderate dose is appropriate in the case of piracy.

Discussion

Our research makes interesting theoretical contributions, and our results have important managerial and policy implications. In this section, we highlight them.

Theoretical Contributions

Prior literature has found that the manufacturer of an information good may find piracy desirable if it enhances consumers' valuation for the legal product, leading to an increase in its overall demand. For example, piracy increases the size of the installed base and, in the presence of a positive network effect, may enhance the overall valuation (Conner and Rumelt 1991). Similarly, when the pirated version of an experience good serves as a product sample for consumers, consumers' valuation for the product may increase upon a positive experience with the pirated version (Chellappa and Shivendu 2005). In both cases, it may be desirable for the manufacturer to tolerate some piracy, as it enhances the demand for the legal product. However, absent any such positive impact on the valuation or demand, prior research has consistently found the manufacturer of an information good to prefer more enforcement and less piracy (Chen and Png 2003; Lahiri and Dey 2013). Interestingly, in our setting, piracy has no positive impact on the demand for the legal product; yet, we find that piracy may not only enhance the profit of the manufacturer but also that of the retailer. That piracy can enhance private profits this way is a completely new insight and adds to the literature on piracy and its impacts.

Of course, if we were to also throw demand enhancements into the mix, the private profits would certainly be larger, and the willingness to tolerate piracy would be even higher. We do find this intuition to be correct. In the previous section, we illustrate this notion by adding a positive network effect. The win–win region actually expands in the presence of the network effect, implying that the manufacturer and retailer are both open to tolerating even more piracy.

Given that our modeling setup is quite similar to the ones used in prior literature, how could we find such a surprising and different result? We do so because we explicitly model the vertical structure of the supply chain in the context of piracy. Prior literature has mostly viewed the supply side as consisting of only one party, typically referred to as the manufacturer. In doing so, it has only looked at the first order impact of piracy as shadow competition; since more competition is undesirable to the manufacturer, piracy has always been an unwanted nuisance. In contrast, we dissect the vertical structure and allow the manufacturer and retailer to act strategically. As a result, the channel may be faced with double marginalization. We find that piracy may play an unlikely role in coordinating the channel and reducing the negative impact of double marginalization. This insight is also new and adds to our understanding of the impacts of piracy.

How does piracy end up coordinating the channel? In essence, the shadow competition from piracy has a first and a second order effect on both the parties. The first order effect is that piracy hurts both the manufacturer and retailer as it takes away a portion of their market power. The second order effect, however, is positive on both of them: when one party is hurt, the other indirectly gains from it.[10] For moderate levels of piracy, the second order effect can dominate the first, resulting in a situation where both parties may be better off.

Now, how is the shadow competition from piracy different from regular up- or down-stream competitions? Downstream competition, for example, can have only the first order effect on a retailer and only the second order effect on the manufacturer. Thus, a retailer dislikes more downstream competition, but the manufacturer welcomes it. Similarly, upstream competition is desired by the retailer, but not by manufacturers. In contrast, the shadow competition of piracy exerts both the first order and second order effects on both parties at the same time, enabling a win–win situation to emerge. Our work contributes to the current literature by unraveling this subtle interplay.

Finally, we not only contribute by finding a counterintuitive impact of piracy, but we also show that our results are reasonably robust. To establish their robustness, in the previous two sections, we relax several key assumptions about the model setup and show that our results remain qualitatively the same: moderate doses of piracy continue to mitigate double marginalization.

Managerial Insights

Our research has interesting managerial implications as well. First, the very idea that piracy can have the unexpected role of better aligning the channel's incentives is new. The notion from Theorem 1 that a moderate level of enforcement is preferable to both parties than a low or high level should give the channel partners a reason to pause before pursuing costly enforcement activities.

This pause to carefully deliberate the unintended consequences of enforcement activities is indeed a necessary one. It is clear from Figure 2 that, when enforcement is low ($r < \rho_3$) and piracy is rampant, both the manufacturer and retailer operate within the lose–lose window, because of severe cannibalization from piracy. Thus, a measured increase in enforcement could push them both to the win–win window and make them happier. However, if they become overzealous in prosecuting illegal downloaders or in lobbying for more enforcement—if the push becomes a shove—they might end up in the win–lose or even the benchmark region where piracy is simply too weak to combat double marginalization effectively. Some moderation is, therefore, warranted. In fact, in

[10]Essentially, when one party is forced to reduce its margin, it automatically grants the other party a higher revenue by means of a higher margin or a higher overall demand, or both.

this regard, the retailer has to be extra careful, as its winning window is narrower than that of the manufacturer.

As a word of caution, our results do not imply that the legal channel should, all of a sudden, start actively encouraging piracy. The implication is simply that, situated in a real-world context, our manufacturer and retailer should recognize that a certain level of piracy or its threat might actually be beneficial and should, therefore, exercise some moderation in their antipiracy efforts. This could manifest itself in them tolerating piracy to a certain level, perhaps by sometimes turning a blind eye to it. Such a strategy would indeed be consistent with how Tassi (2014) describes HBO's "don't-care" attitude toward piracy of its products.

Further, as shown in Figure 6, despite the existence of the win–win window, the optimal enforcement level is quite different for the two parties. Since $r_m^* > r_r^*$, it is clear that the manufacturer always prefers more enforcement than does the retailer. Given this fact, as well as the threat of the adjoining win–lose window (see Figure 2(b)) the retailer may actually be somewhat reluctant to push for higher enforcement. This is indeed borne out by real-world observations. For example, the lawsuits against music pirates brought on by the Recording Industry Association of America (RIAA) were all on behalf of the major record labels (such as EMI, Sony, and Warner), but did not include any retailer as an afflicted party. Similar trends are observed with lawsuits filed by the Motion Pictures Association of America (MPAA) as well.[11]

Finally, why do information-good supply chains not adopt mechanisms that can coordinate the channel? It is a fair question to ask because, after all, if double marginalization is indeed a serious issue for both the parties in the chain, they ought to look for a way to curb its impact. Then, why do they not do something about it? The answer to this question, too, is actually found in our results. Since piracy or its threat can also curb the impact of double marginalization, and since piracy of digital goods is quite common in the real world, perhaps the supply chain feels no urge to do anything extra. Recognizing that piracy can be an antidote to double marginalization and vice versa, the supply chain may simply adopt a lackadaisical, passive approach toward both tasks, those of stopping piracy and coordinating the channel. Viewed this way, our results explain the current realities of many markets: Supply chains dislike piracy and double marginalization, but only in isolation. Had there been no double marginalization, they would have likely taken a stronger stand against piracy. Likewise, had there been no piracy, they would perhaps have used more sophisticated mechanisms to address double marginalization. However, because both piracy and double marginalization exist at the same time, supply chains may actually welcome their coexistence, at least to an extent.

Policy Implications

Our results also shed some light on policy debates surrounding digital piracy. First, over a very large portion of the parameter space, $r \in [0, \rho_5)$, the social surplus is larger in the presence of piracy or its threat than without, indicating the need for a policy that is tolerant of some level of piracy. What is important to note here is that this result does not have to rely on the surplus generated through pirated use: even when the pirate surplus is excluded, the result in Theorem 3 holds as stated. Of special interest is the region (ρ_5, ρ_2), where piracy actually exacerbates the effect of double marginalization, and the social surplus nosedives to a level even below its benchmark value. The message is clear. A higher level of enforcement is not always better and the social planner, if not careful, may choose a level so high that it leads to a paltry social surplus, making the situation quite grim.

Also, a unique aspect of our context is the existence of a win–win–win window, (ρ_3, ρ_4), where all of the parties—manufacturer, retailer, and consumers—are better off, somewhat defying the well-known tension between private profits and public welfare. Stated differently, unlike the results in prior research, our social planner does have the ability to make all of the parties happy at the same time, by adopting a measured approach.

Of course, the socially optimal enforcement level (r_S^*) is still less than what the manufacturer, or even the retailer, desires. A social planner must recognize this fact and should accordingly exercise further moderation when stepping up enforcement. At the same time, though, the planner must also

[11]Notably, such lawsuits have often come under heavy fire: "The individual lawsuits were unbelievably counterproductive. The record companies basically bought themselves a huge amount of bad publicity, a few settlements and no real impact on file-sharing" (Holpuch 2012). Apparently, the RIAA ended up spending $17.6 million in lawsuits only to recoup $391,000 in settlements (Masnick 2010). Despite such criticism, it is difficult to accept that the RIAA and the major record labels were simply acting in an impulsive manner and that the MPAA blindly followed the RIAA example without carefully thinking through the consequences. Indeed, it is possible to argue that the situation was pretty grim for the record labels prior to the lawsuits. At that time, essentially any music ever recorded and released was available on the P2P networks—first Napster, followed by Scour, Aimster, Grokster, Morpheus, Kazaa, and LimeWire, just to name a few—and most music pirates did not even consider it illegal to share copyrighted music. If such copyright violations were allowed to continue unchallenged, the very existence of the record labels could have been at stake. Viewed this way, the lawsuits were likely targeted not at generating additional revenue from the penalty but toward making people aware of this illegality and generating some fear, eventually leading to an elevated r in consumers' minds. Not surprisingly, the RIAA stopped these lawsuits as soon as it generated sufficient publicity, lending further support to this view.

realize that, when the enforcement level is very low and piracy rampant, it can hurt the legal channel so badly that the health of the industry may become a concern. We do see that in our model setting. When r approaches zero, the manufacturer and retailer both suffer heavily, as they make a profit that is well below the benchmark case (see Figure 2(a)). Such low profits could eventually drive the manufacturer out of producing the good for good. Granted that such considerations do not enter our welfare calculations, but then, in public policy, there ought to be considerations well beyond the maximization of a simple social welfare function. A thorough review of those considerations has a certain normative aspect and is naturally beyond the scope of our positive experiment.

Conclusion

In this work, we extend the literature on piracy to contexts of retailer-sold information goods, where double marginalization is potentially a factor. To squarely focus on how this vertical externality interacts with piracy, we develop a parsimonious model. We assume away factors that can make piracy intuitively more appealing from the legal channel's viewpoint and simply narrow down to a supply chain that faces two problems: double marginalization and piracy. Given no obvious beneficial impacts of piracy, one might expect the situation to be quite grave for the legal channel that is now fraught with two concerns. However, as we show, piracy reacts with double marginalization in a rather interesting manner that could lead to higher profits for both the manufacturer and retailer as well as a higher surplus for consumers, resulting in a surprising win–win–win situation. To the best of our knowledge, no other work has viewed piracy in this light and, as a result, all have overlooked this beneficial aspect that ought to make businesses, consumers, and governments rethink the value of antipiracy enforcement.

Our stylized model abstracts away many aspects of the real world, only to stay clear of confounding issues. We later address a few of these aspects, such as consumer heterogeneity, downstream competition, commercial pirates, retailer bundling, possible transfer of penalty to the legal channel, and positive network effects. Our original findings appear robust to these extensions. At the same time, there are a few other aspects that have not been discussed in this work. For example, we do not explicitly model the fact that information goods are often experience goods. Prior research has examined how the ability to sample a pirated copy of an underrated product may lead to a net positive revision of consumers' valuations, resulting in the manufacturer having a more tolerant behavior toward piracy (Chellappa and Shivendu 2005). While we expect our results to extend to such a situation, a careful analysis is certainly warranted, especially because such multi-agent game-theoretic models can often lead to surprises. Nevertheless, this study paves a path for further research into the role of piracy in information-goods markets fraught with vertical externalities.

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References

Abhishek, V., Jerath, K., and Zhang, Z. J. 2016. "Agency Selling or Reselling? Channel Structures in Electronic Retailing," *Management Science* (62:8), pp. 2259-2280.

August, T., and Tunca, T. I. 2008. "Let the Pirates Patch? An Economic Analysis of Network Software Security Patch Restrictions," *Information Systems Research* (19:1), pp. 48-70.

Bae, S.-H., and Choi, J. P. 2006. "A Model of Piracy," *Information Economics and Policy* (18:3), pp. 303-320.

Belleflamme, P., and Peitz, M. 2010. *Industrial Organization: Markets and Strategies*, Cambridge, UK: Cambridge University Press.

Cabral, L. M. B. 2017. *Introduction to Industrial Organization*, Cambridge, MA: The MIT Press.

Cachon, G. P. 2003. "Supply Chain Coordination with Contracts," in *Handbooks in Operations Research and Management Science* (Volume 11), S. C. Graves and . G. de Kok (eds.), Amsterdam: Elsevier, pp. 227-339.

Cachon, G. P., and Lariviere, M. A. 2005. "Supply Chain Coordination with Revenue-Sharing Contracts: Strengths and Limitations," *Management Science* (51:1), pp. 30-44.

Caves, R. E. 2005. *Switching Channels: Organization and Change in TV Broadcasting*, Cambridge, MA: Harvard University Press.

Chellappa, R. K., and Shivendu, S. 2003. "Pay Now or Pay Later? Managing Digital Product Supply Chains," *Proceedings of the 5th International Conference on Electronic Commerce*, pp. 230-234.

Chellappa, R. K., and Shivendu, S. 2005. "Managing Digital Piracy: Pricing and Sampling Strategies for Digital Experience Goods in Vertically Segmented Markets," *Information Systems Research* (16:4), pp. 400-417.

Chen, Y.-N., and Png, I. 2003. "Information Goods Pricing and Copyrights Enforcement: Welfare Analysis," *Information Systems Research* (14:1), pp. 107-123.

Conner, K. R., and Rumelt, R. O. 1991. "Software Piracy: An Analysis of Protection of Strategies," *Management Science* (37:2), pp. 125-139.

Crawford, G. S., Lee, R. S., Whinston, M. D., and Yurukoglu, A. 2017. "The Welfare Effects of Vertical Integration in Multichannel Television Markets," NBER Working Paper No. 21832 (http://www.nber.org/papers/w21832.pdf).

Danaher, B., Dhanasobhon, S., Smith, M. D., and Telang, R. 2010. "Converting Pirates Without Cannibalizing Purchasers: The

Impact of Digital Distribution on Physical Sales and Internet Piracy," *Marketing Science* (29:6), pp. 1138-1151.
Danaher, B., and Smith, M. D. 2014. "Gone in 60 Seconds: The Impact of the MegaUpload Shutdown on Movie Sales," *International Journal of Industrial Organization* (33), pp. 1-8.
Danaher, B., Smith, M. D., Telang, R., and Chen, S. 2014. "The Effect of Graduated Response Antipiracy Laws on Music Sales: Evidence from an Event Study in France," *The Journal of Industrial Economics* (62:3), pp. 541-553.
Gil, R., and Warzynski, F. 2015. "Vertical Integration, Exclusivity, and Game Sales Performance in the U.S. Video Game Industry," *Journal of Law, Economics, and Organization* (31:S1), pp. 143-168.
Gopal, R. D., and Gupta, A. 2010. "Trading Higher Software Piracy for Higher Profits: The Case of Phantom Piracy," *Management Science* (56:11), pp. 1946-1962.
Hao, L., and Fan, M. 2014. "An Analysis of Pricing Models in the Electronic Book Market," *MIS Quarterly* (38:4), pp. 1017-1032.
Höhn, M. I. 2010. *Relational Supply Contracts*, Berlin: Springer-Verlag.
Holpuch, A. 2012. "Minnesota Woman to Pay $220,000 Fine for 24 Illegally Downloaded Songs," *Guardian* (http://www.guardian.co.uk/technology/2012/sep/11/minnesota-woman-songs-illegally-downloaded; accessed May 8, 2013).
Jain, S. 2008. "Digital Piracy: A Competitive Analysis," *Marketing Science* (27:4), pp. 610-626.
Jaisingh, J. 2009. "Impact of Piracy on Innovation at Software Firms and Implications for Piracy Policy," *Decision Support Systems* (46:4), pp. 763-773.
Jeong, B., Khouja, M., Zhao, K. 2012. "The Impacts of Piracy and Supply Chain Contracts on Digital Music Channel Performance," *Decision Support Systems* (52:3), pp. 590-603.
Johar, M., Kumar, N., and Mookerjee, V. 2012. "Content Provision Strategies in the Presence of Content Piracy," *Information Systems Research* (23:3), pp. 960-975.
Kannan, K., Rahman, M., and Tawarmalani, M. 2016. "Economic and Policy Implications of Restricted Patch Distribution," *Management Science* (62:11), pp. 3161-3182.
Karaganis, J. 2011. "Rethinking Piracy," in *Media Piracy in Emerging Economies*, J. Karaganis (ed.), Brooklyn, NY: Social Science Research Council, pp. 1-73 (http://piracy.ssrc.org/the-report/; accessed May 8, 2013).
Khouja, M., and Park, S. 2007. "Optimal Pricing of Digital Experience Goods Under Piracy," *Journal of Management Information Systems* (24:3), pp. 109-141.
Lahiri, A. 2012. "Revisiting the Incentive to Tolerate Illegal Distribution of Software Products," *Decision Support Systems* (53:2), pp. 357-367.
Lahiri, A., and Dey, D. 2013. "Effects of Piracy on Quality of Information Goods," *Management Science* (59:1), pp. 245-264.
Masnick, M. 2010. "RIAA Spent $17.6 Million in Lawsuits... to Get $391,000 in Settlements?," *Techdirt* (https://www.techdirt.com/articles/20100713/17400810200.shtml; accessed April 19, 2017).
Morris, C. 2016. "Blu-ray Struggles in the Streaming Age," *Fortune* (http://fortune.com/2016/01/08/blu-ray-struggles-in-the-streaming-age/; accessed May 4, 2017).
Premkumar, G. P. 2003. "Alternate Distribution Strategies for Digital Music," *Communications of the ACM* (46:9), pp. 89-95.
Smith, A. 1776. *The Wealth of Nation*, Amherst, NY: Prometheus Book.
Spengler, J. J. 1950. "Vertical Integration and Antitrust Policy," *Journal of Political Economy* (58:4), pp. 347-352.
Sundararajan, A. 2004. "Managing Digital Piracy: Pricing and Protection," *Information Systems Research* (15:3), pp. 287-308.
Tassi, P. 2014. "'Game of Thrones' Sets Piracy World Record, but Does HBO Care?," *Forbes* (http://www.forbes.com/sites/insertcoin/2014/04/15/game-of-thrones-sets-piracy-world-record-but-does-hbo-care/; accessed October 23, 2014).
Tirole, J. 1992. *The Theory of Industrial Organization*, Cambridge, MA: The MIT Press.
Tunca, T. I., and Wu, Q. 2013. "Fighting Fire with Fire: Commercial Piracy and the Role of File Sharing on Copyright Protection Policy for Digital Goods," *Information Systems Research* (24:2), pp. 436-453.
Van der Sar, E. 2017. "Game of Thrones Piracy Peaks after Season Finale," *TorrentFreak* (https://torrentfreak.com/game-of-thrones-piracy-peaks-after-season-finale-170828/; accessed October 13, 2017).
Vernik, D. A., Purohit, D., and Desai, P. S. 2011. "Music Downloads and the Flip Side of Digital Rights Management," *Marketing Science* (30:6), pp. 1011-1027.
Wu, S.-Y., and Chen, P.-Y. 2008. "Versioning and Piracy Control for Digital Information Goods," *Operations Research* (56:1), pp. 157-172.

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THE "INVISIBLE HAND" OF PIRACY: AN ECONOMIC ANALYSIS OF THE INFORMATION-GOODS SUPPLY CHAIN

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Appendix A

Technical Details of All the Extensions

In this appendix, we provide some of the technical details that were omitted from the main paper for readability.

Heterogeneity in Piracy Cost

Given the demand function in (3), we can solve the retailer's maximization problem $\max_p (p-w)q(p)$

$$p(w)=\begin{cases} p_0(w)=\frac{1+w}{2}, & \text{Case R0}\\ p_1(w)=\frac{1-\beta+r(1-\alpha)+w(1-\alpha\beta)}{2(1-\alpha\beta)} & \text{Case R1}\\ p_2(w)=\frac{1+w}{2}, & \text{Case R2}\\ p_3(w)=\frac{1-\beta+r(1-\alpha)+\alpha(g+hr)+w}{2}, & \text{Case R3}\\ p_4(w)=\frac{1-\beta+g+hr+w}{2}, & \text{Case R4}\end{cases}$$

For each case, we can now solve the manufacturer's profit maximization problem $\max_{w_i} w_i p_i(w_i)$ to obtain the wholesale price, which can be substituted above to get the retail price. The overall solution for each case can be written as

$$(w,p)=\begin{cases} (w_0,p_0)=\left(\frac{1}{2},\frac{3}{4}\right), & \text{Case R0}\\ (w_1,p_1)=\left(\frac{1-\beta+r(1-\alpha)}{2(1-\alpha\beta)},\frac{3(1-\beta+r(1-\alpha))}{4(1-\alpha\beta)}\right), & \text{Case R1}\\ (w_2,p_2)=\left(\frac{1}{2},\frac{3}{4}\right), & \text{Case R2}\\ (w_3,p_3)=\left(\frac{1-\beta+r(1-\alpha)+\alpha(g+hr)}{2},\frac{3(1-\beta+r(1-\alpha)+\alpha(g+hr))}{4}\right), & \text{Case R3}\\ (w_4,p_4)=\left(\frac{1-\beta+g+hr}{2},\frac{3(1-\beta+g+hr)}{4}\right), & \text{Case R4}\end{cases}$$

We now turn our attention to the limit regions:

- **Case R1A:** In this region, the retailer is forced to set $p_{1\mathrm{A}} = \frac{r}{\beta}$. The wholesale price in this case is $w_{1\mathrm{A}} = \frac{r}{\beta} - \frac{(1-\beta)(\beta-r)}{\beta(1-\alpha\beta)}$, which can be found by simply equating $p_1(w)$ to $p_{1\mathrm{A}}$ and solving for w.
- **Case R1B:** In this region, the retailer must set $p_{1\mathrm{B}} = 1-\beta+r$. The wholesale price in this case is given by $w_{1\mathrm{B}} = 2(1-\beta) + r - \frac{(1-\beta)(1-r\alpha)}{1-\alpha\beta}$, which is the solution of $p_1(w) = p_{1\mathrm{B}}$.
- **Case R3A:** In this region, too, the retailer is forced to set $p_{3\mathrm{A}} = 1-\beta+r$. The corresponding wholesale price is obtained from the solution of $p_3(w) = p_{3\mathrm{A}}$ and is given by $w_{3\mathrm{A}} = 1-\beta+r-\alpha(g+hr-r)$.
- **Case R3B:** The limit retail price in this case is given by $p_{3\mathrm{B}} = \frac{g+hr}{\beta}$. The corresponding wholesale price is obtained from the solution of $p_3(w) = p_{3\mathrm{B}}$ and is given by $w_{3\mathrm{B}} = \frac{(2-\alpha\beta)(g+hr)}{\beta} - (1-\beta+r(1-\alpha))$. In this case, a valid retail price must satisfy $\frac{p_{3\mathrm{B}}-r}{1-\beta} \leq 1$.
- **Cases R4A and R4B:** In these cases as well, the retailers is forced to set a limit retail price of $p_{4\mathrm{A}} = p_{4\mathrm{B}} = \frac{g+hr}{\beta}$. The wholesale price in R4A is obtained from the solution of $p_3(w) = p_{4\mathrm{A}}$ and is given by $w_{4\mathrm{A}} = \frac{(2-\alpha\beta)(g+hr)}{\beta} - (1-\beta+r(1-\alpha))$, the only difference with R3B being that, now, $\frac{p_{4\mathrm{A}}-r}{1-\beta} > 1$. Case 4A must also satisfy $w_{4\mathrm{A}} < p_{4\mathrm{A}}$. When this is violated, we enter Case 4B as another limit case, where $w_{4\mathrm{A}} = p_{4\mathrm{A}} = \frac{g+hr}{\beta}$.

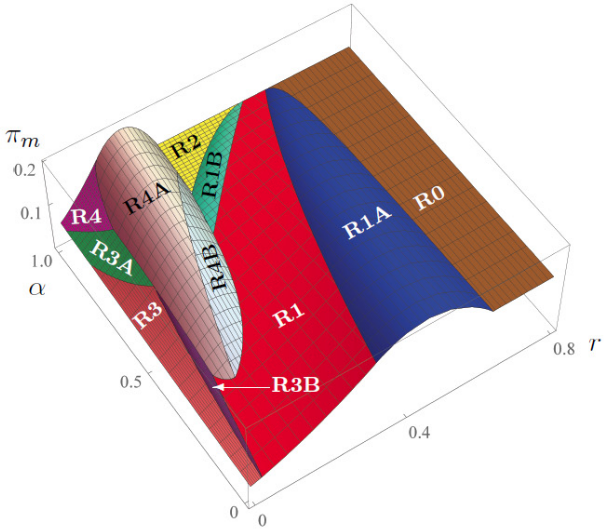
With these closed form solutions for wholesale and retail prices, it is easy to find the manufacturer's and retailer's profits as

$$(\pi_m, \pi_r) = \begin{cases} (\pi_{m0}, \pi_{r0}) = \left(\frac{1}{8}, \frac{1}{16}\right), & \text{Case R0} \\ (\pi_{m1}, \pi_{r1}) = \left(\frac{(1-\beta+r(1-\alpha))^2}{8(1-\beta)(1-\alpha\beta)}, \frac{(1-\beta+r(1-\alpha))^2}{16(1-\beta)(1-\alpha\beta)}\right), & \text{Case R1} \\ (\pi_{m1\mathrm{A}}, \pi_{r1\mathrm{A}}) = \left(\frac{(\beta-r)\left(r(2-\beta(1+\alpha))-\beta(1-\beta)\right)}{\beta^2(1-\alpha\beta)}, \frac{(\beta-r)^2(1-\beta)}{\beta^2(1-\alpha\beta)}\right), & \text{Case R1A} \\ (\pi_{m1\mathrm{B}}, \pi_{r1\mathrm{B}}) = \left(\frac{\alpha(\beta-r)(1-\beta+r(1+\alpha)-2\alpha\beta(1-\beta+r))}{1-\alpha\beta}, \frac{\alpha^2(\beta-r)^2(1-\beta)}{1-\alpha\beta}\right), & \text{Case R1B} \\ (\pi_{m2}, \pi_{r2}) = \left(\frac{\alpha}{8}, \frac{\alpha}{16}\right), & \text{Case R2} \\ (\pi_{m3}, \pi_{r3}) = \left(\frac{(1-\beta+r(1-\alpha)+\alpha(g+hr))^2}{8(1-\beta)}, \frac{(1-\beta+r(1-\alpha)+\alpha(g+hr))^2}{16(1-\beta)}\right), & \text{Case R3} \\ (\pi_{m3\mathrm{A}}, \pi_{r3\mathrm{A}}) = \left(\frac{\alpha(g+hr-r)(1-\beta-\alpha(g+hr)+r(1+\alpha))}{1-\beta}, \frac{\alpha^2(g+hr-r)^2}{1-\beta}\right), & \text{Case R3A} \\ (\pi_{m3\mathrm{B}}, \pi_{r3\mathrm{B}}), & \text{Case R3B} \\ (\pi_{m4}, \pi_{r4}) = \left(\frac{\alpha(1-\beta+g+hr)^2}{8(1-\beta)}, \frac{\alpha(1-\beta+g+hr)^2}{16(1-\beta)}\right), & \text{Case R4} \\ (\pi_{m4\mathrm{A}}, \pi_{r4\mathrm{A}}), & \text{Case R4A} \\ (\pi_{m4\mathrm{B}}, \pi_{r4\mathrm{B}}) = \left(\frac{\alpha(g+hr)(\beta-g-hr)}{\beta^2}, 0\right), & \text{Case R4B} \end{cases}$$

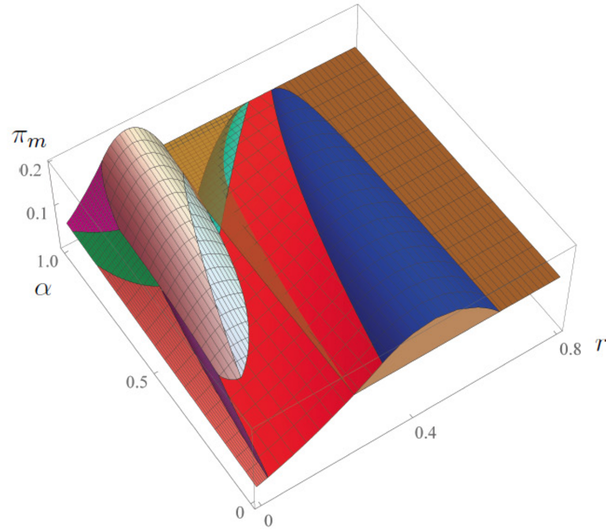
where

$$\pi_{m3\mathrm{B}} = \frac{(2(g+hr)-\beta(1-\beta+r(1-\alpha)+\alpha(g+hr)))(\beta(1-\beta+r(1-\alpha))-(g+hr)(1-\alpha\beta))}{\beta^2(1-\beta)},$$
$$\pi_{r3\mathrm{B}} = \frac{(g+hr-\beta(1-\beta+r(1-\alpha)+\alpha(g+hr)))^2}{\beta^2(1-\beta)},$$
$$\pi_{m4\mathrm{A}} = \frac{\alpha(g+hr-\beta)(\beta(1-\beta+r(1-\alpha)+\alpha(g+hr))-2(g+hr))}{\beta^2}, \quad \text{and}$$
$$\pi_{r4\mathrm{A}} = \frac{\alpha(g+hr-\beta)((1-\alpha\beta)(g+hr)-\beta(1-\beta+r(1-\alpha)))}{\beta^2}.$$

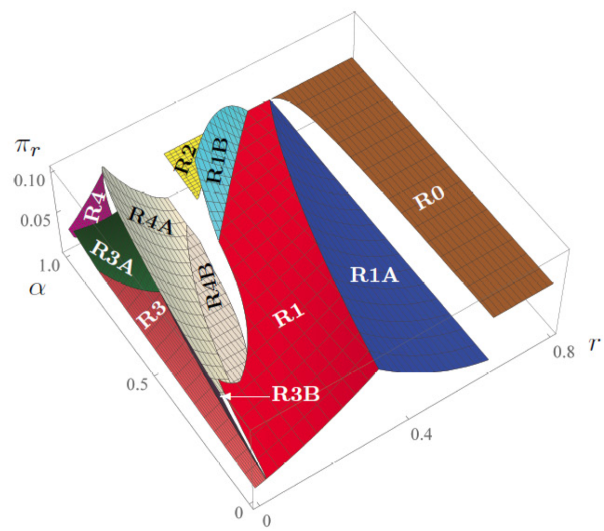
The boundaries between these regions are obtained in two steps. First, we apply the validity conditions in (3) to R0, R1, R2, R3, and R4. We also apply the appropriate validity conditions to all the six limit regions. Once we have curtailed these individual regions by their validity conditions, only a few overlapping regions remain. To determine their explicit boundaries, we then compare the manufacturer's profits across those overlapping cases. Because all our price and profit expressions are in closed form, we can easily find these boundaries in closed form as well. Once we curtail the overlapping regions using these boundaries, we get a unique equilibrium solution for every point in the parameter space. We omit the cumbersome algebraic expressions in favor of plots of the manufacturer's and retailer's profits as functions of r and α; Figure A1 shows these profit plots; for these plots, $\beta = 0.75$, and the heterogeneity level is moderate ($g = 0.1$ and $h = 2$). It is comforting to see that a two-dimensional slice of these plots for very small α-values mimic our results depicted in Figure 2(a).



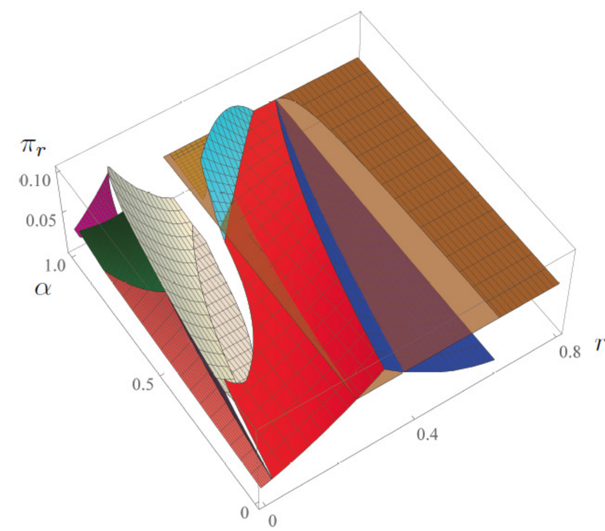
(a) Manufacturer's Profit



(b) Manufacturer's Profit vs. Benchmark



(c) Retailer's Profit



(d) Retailer's Profit vs. Benchmark

Figure A1. Profit as a Function of r and α; $\beta = 0.75, g = 0.1, h = 2$

A careful observation of the plots in Figure A1(b) and (d) reveals that there is indeed a region spanning portions of R1 and R1A, where both the manufacturer and retailer have profits higher than their respective benchmark values in R0. In fact, the red-blue humps in both plots over the translucent R0-plane are clearly visible. This win-win region is denoted by $(\tilde{\rho}_3, \tilde{\rho}_4)$ in Figure 7; $\tilde{\rho}_3$ is obtained by comparing π_{m1} and π_{m1B} with π_{m0}, and $\tilde{\rho}_4$, by comparing π_{r1A} with π_{r0}. We find

$$\tilde{\rho}_3 = \begin{cases} \dfrac{\sqrt{(1-\beta)(1-\alpha\beta)} - (1-\beta)}{1-\alpha}, & \text{if } \alpha \leq \frac{7-8\beta+\sqrt{49-48\beta}}{32\beta(1-\beta)} \\ \dfrac{\sqrt{2\alpha(1-\alpha)(1-\alpha\beta)(2\alpha\beta-1)}}{4\alpha(\alpha(2\beta-1)-1)} + \dfrac{1-2\beta-3\alpha\beta+4\alpha\beta^2}{2(\alpha(2\beta-1)-1)}, & \text{otherwise} \end{cases}$$

and

$$\tilde{\rho}_4 = \beta\left(1 - \frac{1}{4}\sqrt{\frac{1-\alpha\beta}{1-\beta}}\right)$$

A point to note here is that the above thresholds are independent of both g and h, and depend only on α, the fraction of the high type. Furthermore, in the case of no heterogeneity, that is, when $\alpha \to 0$, they reduce to the original (ρ_3, ρ_4) window

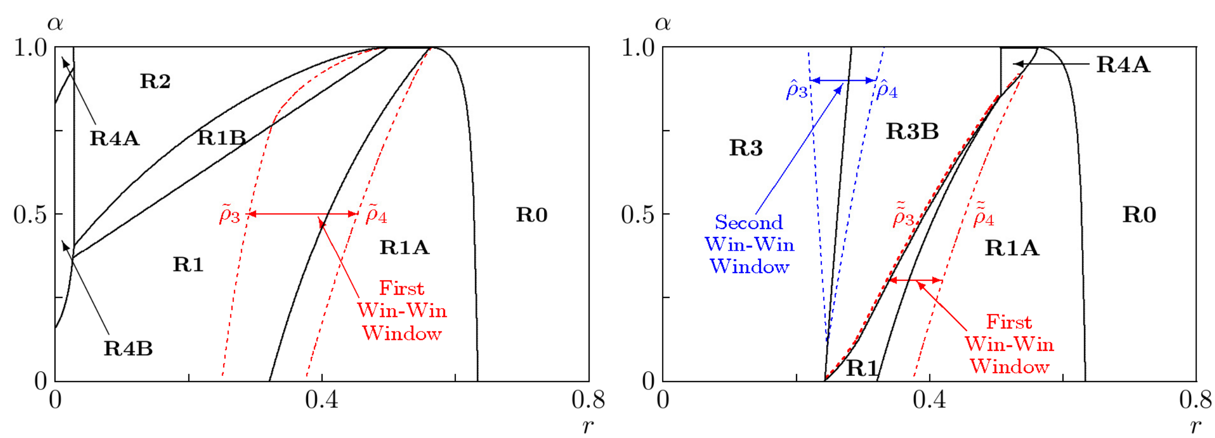
$$\lim_{\alpha\to 0} \tilde{\rho}_3 = \rho_3 \quad \text{and} \quad \lim_{\alpha\to 0} \tilde{\rho}_4 = \rho_4.$$

A structural observation is now in order. There are essentially two levers that control heterogeneity in the piracy cost. The first lever, α, which simply indicates the *extent of heterogeneity*, exhibits a behavior that is essentially the same at both the extremes. When α is small, we get back our original situation, because the fraction of the high type is negligible, making heterogeneity disappear for all practical purposes.

However, the same is also true for very high α, in which case, the fraction of the low type is negligible, and we get back our original problem with a linearly transformed piracy cost. This inherent symmetry of the setup is quite important to fully grasp this complicated analysis. Now, while α indicates the extent, the *level of heterogeneity* is determined by the second lever of the (g, h) pair—when g and h are high, either individually or together, heterogeneity is high, but, when they are both small, that is, when $g \to 0$ and $h \to 1$, heterogeneity once again disappears, and we get back to our original problem setting.

Now, even though the $(\tilde{\rho}_3, \tilde{\rho}_4)$ window is independent of g and h, we are still not assured of the existence of a win-win window. To fully understand the impact of g and h on the existence of the win-win window, we need to determine what happens when they move from their moderate values of $g = 0.1$ and $h = 2$ as reported in Figure A1. It turns out that the $\tilde{\rho}_3$-threshold, which was obtained by comparing π_{m1} and π_{m1B} with π_{m0}, may no longer provide the valid left limit of the win-win window, if boundaries of R1 and R1B encroach upon $\tilde{\rho}_3$.

When g or h increases from its moderate value, there are no problems with the win-win window represented by $(\tilde{\rho}_3, \tilde{\rho}_4)$. This is because the regions to the left of R1 and R1B actually move further to the left when either g or h increases. Therefore, there is no encroaching on $\tilde{\rho}_3$, and the win-win window derived above remains intact. This is clearly visible in Figure A2(a).



(a) High Heterogeneity: g=0.5, h=5 (b) Low Heterogeneity: g=0.01 and h=1.1

Figure A2. Partitions of the (r, α) Space for Extreme g and h; $\beta = 0.75$

However, as both g and h become small, the regions to the left of R1 and R1B start moving in towards the right, squeezing R1 and R1B in the process. At some point, when g and h are both really small, the boundary between R1 and R4B moves in sufficiently to encroach on the $\tilde{\rho}_3$-threshold; see Figure A2(b). When that happens, $(\tilde{\rho}_3, \tilde{\rho}_4)$ is no longer the valid win-win window. The correct one becomes $(\tilde{\tilde{\rho}}_3, \tilde{\tilde{\rho}}_4)$

$$\tilde{\tilde{\rho}}_3 = \max\{\tilde{\rho}_3, b_1, b_2, b_3\} \quad \text{and} \quad \tilde{\tilde{\rho}}_4 = \max\{\tilde{\rho}_4, b_3\}$$

where b_1 is the boundary between regions R1 and R4B, b_2 between R1 and R3B, and b_3 between R1A and R4A. When g and h are very small, all these boundaries, b_1, b_2, and b_3, get pushed to the right, resulting in some shrinkage of the win-win window, $(\tilde{\tilde{\rho}}_3, \tilde{\tilde{\rho}}_4)$. However, well before this win-win window can be fully usurped, a second win-win window starts appearing to its left. The emergence of this second win-win window may seem surprising at first, but can be clearly predicted from the symmetry of the problem we discussed earlier. The first win-win window, $(\tilde{\tilde{\rho}}_3, \tilde{\tilde{\rho}}_4)$, occurs because of the existence of the low type. When the *level* of heterogeneity is low, that is, both g and h are small, the high type is now very close to the low type and must, therefore, behave in a similar fashion, implying that the high type ought to get a win-win window of its own.

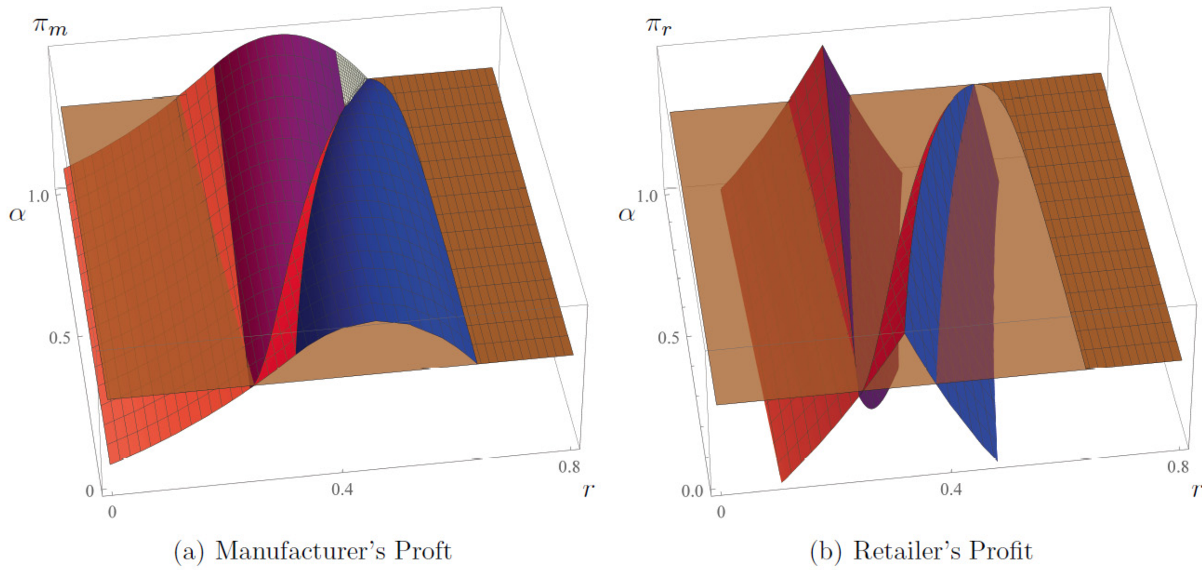


Figure A3. Profit as a Function of r and α; $\beta = 0.75, g = 0.01, h = 1.1$

To illustrate, we once again plot the manufacturer's and retailer's profits in Figure A3, this time for $g = 0.01$ and $h = 1.1$. Figure A3 clearly reveals the pink-purple humps above the benchmark levels in both the profit plots; of course, these humps are there in addition to the original red-blue ones, which have now shrunk somewhat. This second win-win window is denoted $(\hat{\rho}_3, \hat{\rho}_4)$; $\hat{\rho}_3$ and $\hat{\rho}_4$ can be easily obtained by comparing the retailer's profit in regions R3 and R3B with their benchmark value in R0. We get

$$\hat{\rho}_3 = \frac{\sqrt{1-\beta}-(1-\beta+g\alpha)}{1+\alpha(h-1)} \quad \text{and} \quad \hat{\rho}_4 = \frac{\beta\left(4(1-\beta)-\sqrt{1-\beta}\right)-4g(1-\alpha\beta)}{4(h(1-\alpha\beta)-\beta(1-\alpha))}$$

As g and h decrease even further, the first window, $(\tilde{\tilde{\rho}}_3, \tilde{\tilde{\rho}}_4)$, shrinks, but the second window, $(\hat{\rho}_3, \hat{\rho}_4)$, actually expands. It is easy to see that, when heterogeneity is absent, the second window becomes the same as the original (ρ_3, ρ_4) window, because

$$\lim_{\substack{g\to 0\\ h\to 1}} \hat{\rho}_3 = \rho_3 \quad \text{and} \quad \lim_{\substack{g\to 0\\ h\to 1}} \hat{\rho}_4 = \rho_4$$

Commercial Pirates

In this setup, a consumer can enjoy a utility of $(v-p)$ from purchasing the legal version, or $(v\beta - r - s)$ from a pirated copy. Similar to (1), the legal and illegal demands for given p and s, respectively denoted $q(p,s)$ and $\bar{q}(p,s)$, can now be rewritten as

$$q(p) = \begin{cases} 1 - \frac{p-(r+s)}{1-\beta}, & \text{if } p > \frac{r+s}{\beta} \\ 1-p, & \text{otherwise} \end{cases} \quad \text{and} \quad \bar{q}(p) = \begin{cases} \frac{p-(r+s)}{1-\beta} - \frac{r+s}{\beta}, & \text{if } p > \frac{r+s}{\beta} \\ 0, & \text{otherwise} \end{cases} \qquad \text{(A1)}$$

Given these demand functions, the commercial pirate chooses s in order to maximize its profit $\pi_s(s) = s\,\bar{q}(p,s)$. Since $\frac{\partial^2 \pi_s}{\partial s^2} = -\frac{2}{\beta(1-\beta)} < 0$, we solve the first order condition, $\frac{\partial \pi_s}{\partial s} = \frac{p\beta - r - 2s}{\beta(1-\beta)} = 0$, to obtain the optimal s for a given p

$$s^*(p) = \begin{cases} \frac{p\beta - r}{2}, & \text{if } p > \frac{r}{\beta} \\ 0, & \text{otherwise} \end{cases} \qquad \text{(A2)}$$

Anticipating this response from the commercial pirate, the retailer chooses p in order to maximize its profit $\pi_r(p) = (p - w)\, q(p, s^*(p))$. Now, if $q(p,s) = 1 - \frac{p-(r+s)}{1-\beta}$, then $\pi_r(p,s) = (p-w)\left(1 - \frac{p-(r+s)}{1-\beta}\right)$. Substituting s for $s^*(p)$ in (A2) and taking the derivative with respect to p, we obtain

$$\frac{\partial \pi_r}{\partial p} = \frac{2+r+2w-2p(2-\beta)-\beta(2+w)}{2(1-\beta)} \tag{A3}$$

Since $\frac{\partial^2 \pi_r}{\partial p^2} = -1 - \frac{1}{1-\beta} < 0$, the first-order condition results in $p^*(w) = \frac{r+2(1+w)-\beta(2+w)}{2(2-\beta)}$, which, according to (A1), must be greater than $\frac{r+s^*(p^*(w))}{\beta}$, or $w > \frac{2r}{\beta} - 1 + \frac{\beta-r}{2-\beta}$, for this solution to be valid.

If, on the other hand, $q(p,s) = 1 - p$, then $\pi_r(p,s) = \pi_r(p) = (p-w)(1-p)$. Therefore, we get

$$\frac{\partial \pi_r}{\partial p} = 1 - 2p + w \tag{A4}$$

Since the second-order condition is trivially satisfied, we can equate (A4) to zero to obtain $p^*(w) = \frac{1+w}{2}$, which must be smaller than $\frac{r+s^*(p^*(w))}{\beta}$, or $w < \frac{2r}{\beta} - 1$, for this solution to be valid.

Now, for moderate values of w, that is, if $\frac{2r}{\beta} - 1 \le w \le \frac{2r}{\beta} - 1 + \frac{\beta-r}{2-\beta}$, $\frac{\partial \pi_r}{\partial p}$ given by (A3) is negative, whereas that given by (A4) is positive. Naturally, the optimal p is simply $\frac{r}{\beta}$. Taken together, the optimal retail price for a given w, $p^*(w)$, is

$$p^*(w) = \begin{cases} \frac{r+2(1+w)-\beta(2+w)}{2(2-\beta)}, & \text{if } w > \frac{2r}{\beta} - 1 + \frac{\beta-r}{2-\beta} \\ \frac{r}{\beta}, & \text{if } \frac{2r}{\beta} - 1 \le w \le \frac{2r}{\beta} - 1 + \frac{\beta-r}{2-\beta} \\ \frac{1+w}{2}, & \text{otherwise} \end{cases} \tag{A5}$$

The manufacturer, the first mover in the game, anticipates the retailer's pricing decisions and chooses the optimal wholesale price w^* to maximize $\pi_m(w) = w\, q(p^*(w), s^*(p^*(w)))$. It is clear from (A5) that we have three cases to consider: (i) $w > \frac{2r}{\beta} - 1 + \frac{\beta-r}{2-\beta}$, (ii) $\frac{2r}{\beta} - 1 \le w \le \frac{2r}{\beta} - 1 + \frac{\beta-r}{2-\beta}$, and (iii) $w < \frac{2r}{\beta} - 1$.

For case (i), the manufacturer's profit is $\pi_m = \frac{w(2+r-2\beta-w(2-\beta))}{4(1-\beta)}$. Since $\frac{\partial^2 \pi_m}{\partial w^2} = -\frac{2-\beta}{2(1-\beta)} < 0$, the first order condition, $\frac{\partial \pi_m}{\partial w} = \frac{2+r-2\beta-2w(2-\beta)}{4(1-\beta)} = 0$, results in $w^* = \frac{2+r-2\beta}{2(2-\beta)}$, which, according to (A5), must be greater than $\frac{2r}{\beta} - 1 + \frac{\beta-r}{2-\beta}$, or $r \le \frac{6\beta(1-\beta)}{8-7\beta} = \tilde{\rho}_1$, for this equilibrium to be valid.

For case (ii), $p^* = \frac{r}{\beta}$. The manufacturer, unwilling to leave money on the table, always chooses the highest value from the range $\frac{2r}{\beta} - 1 \le w \le \frac{2r}{\beta} - 1 + \frac{\beta-r}{2-\beta}$, resulting in $w^* = \frac{2r}{\beta} - 1 + \frac{\beta-r}{2-\beta}$. This equilibrium is valid across all $r \le \beta$ since $s^* = 0$ in this equilibrium. If $r > \beta$, then $p^* = \frac{r}{\beta} > 1$, and no consumer would buy the product. Therefore, $r > \beta$ cannot happen in case (ii).

Finally, in case (iii), $p^*(w) = \frac{1+w}{2}$, and the manufacturer's profit is $\pi_m = \frac{w(1-w)}{2}$, implying $w^* = \frac{1}{2}$. According to (A5), this w^* must be no more than $\frac{2r}{\beta} - 1$, implying $r \ge \frac{3\beta}{4} = \tilde{\rho}_5$. It is easy to verify that $\tilde{\rho}_1 < \tilde{\rho}_5$.

Now, case (ii) is the only valid equilibrium if $\tilde{\rho}_1 \le r \le \tilde{\rho}_5$. On the other hand, if $r < \tilde{\rho}_1$, both cases (i) and (ii) are valid. However, the optimal profit from an interior solution ought to be higher, which immediately implies that case (i) is the equilibrium outcome for $r < \tilde{\rho}_1$. Further, if $r > \tilde{\rho}_5$, both cases (ii) and (iii) are possible, and we must compare the manufacturer's profit in these two cases to determine the equilibrium. We can obtain the optimal profits for these two cases using the w^* for the respective cases. The optimal profit for case (ii) is $\frac{(\beta-r)(r(4-3\beta)-2\beta(1-\beta))}{\beta^2(2-\beta)}$, and that for case (iii) is simply $\frac{1}{8}$. Comparing these two profits, it is easy to verify that the manufacturer would choose the first option if $r \ge \frac{\beta(12-10\beta+\sqrt{2\beta(2-\beta)})}{4(4-3\beta)} = \tilde{\rho}_2$. Since $\tilde{\rho}_2 > \tilde{\rho}_5$ holds trivially, (iii) is the equilibrium outcome only if $r \ge \tilde{\rho}_2$.

Combining the above with (A5), the optimal w and p are given by

$$w^* = \begin{cases} \frac{2(1-\beta)+r}{2(2-\beta)}, & \text{if } r < \tilde{\rho}_1 \\ \frac{2r}{\beta} - 1 + \frac{\beta - r}{2-\beta}, & \text{if } \tilde{\rho}_1 \le r < \tilde{\rho}_2 \\ w_0 = \frac{1}{2}, & \text{otherwise} \end{cases} \quad \text{and} \quad p^* = \begin{cases} \frac{6(1-\beta)+3r}{4(2-\beta)}, & \text{if } r < \tilde{\rho}_1 \\ \frac{r}{\beta}, & \text{if } \tilde{\rho}_1 \le r < \tilde{\rho}_2 \\ p_0 = \frac{3}{4}, & \text{otherwise} \end{cases}$$

Using these p^* and w^*, we can find the equilibrium profits for the manufacturer and retailer as π_m^* and π_r^*, respectively:

$$\pi_m^* = \begin{cases} \frac{(2(1-\beta)+r)^2}{16(2-\beta)(1-\beta)}, & \text{if } r < \tilde{\rho}_1 \\ \frac{(\beta-r)(r(4-3\beta)-2\beta(1-\beta))}{\beta^2(2-\beta)}, & \text{if } \tilde{\rho}_1 \le r < \tilde{\rho}_2 \\ \pi_{m0} = \frac{1}{8}, & \text{otherwise} \end{cases} \quad \text{and} \quad \pi_r^* = \begin{cases} \frac{(2(1-\beta)+r)^2}{32(2-\beta)(1-\beta)}, & \text{if } r < \tilde{\rho}_1 \\ \frac{2(1-\beta)(\beta-r)^2}{\beta^2(2-\beta)}, & \text{if } \tilde{\rho}_1 \le r < \tilde{\rho}_2 \\ \pi_{r0} = \frac{1}{16}, & \text{otherwise} \end{cases}$$

We now examine to see if and when the manufacturer and the retailer are better off with piracy than without. First, since in the piracy region $(r < \tilde{\rho}_1)$, the manufacturer's profit, $\pi_m^* = \frac{(2(1-\beta)+r)^2}{16(2-\beta)(1-\beta)}$, is increasing in r, equating this profit to the benchmark profit of $\pi_{m0} = \frac{1}{8}$ and solving for r, we find $r = \sqrt{4 - 2\beta(3-\beta)} - 2(1-\beta)$; of course, for it to be a valid root this r must abide by the restriction, $r < \tilde{\rho}_1$, which is equivalent to $\beta < \frac{16}{17}$. Next, in the threat region $(\tilde{\rho}_1 \le r < \tilde{\rho}_2)$, the manufacturer's profit, $\pi_m^* = \frac{(\beta-r)(r(4-3\beta)-2\beta(1-\beta))}{\beta^2(2-\beta)}$, can never be less than π_{m0}. In other words, for all $\beta \le \frac{16}{17}$, a necessary and sufficient for the manufacturer to be better off is $\sqrt{4-2\beta(3-\beta)} - 2(1-\beta) < r < \tilde{\rho}_2$.

The case of $\beta > \frac{16}{17}$ is somewhat different. Here, the threat region takes over at a lower r; the profit function for the threat region meets the benchmark profit, $\pi_{m0} = \frac{1}{8}$, two times, first at point $\tilde{\rho}_2^c$ and then again at $\tilde{\rho}_2$, where $\tilde{\rho}_2^c$ is the root conjugate to $\tilde{\rho}_2$ and is given by

$$\tilde{\rho}_2^c = \frac{\beta\left(12 - 10\beta - \sqrt{2\beta(2-\beta)}\right)}{4(4-3\beta)}$$

Therefore, for all $\beta > \frac{16}{17}$, the manufacturer would be better off if and only if $\tilde{\rho}_2^c < r < \tilde{\rho}_2$. Defining

$$\tilde{\rho}_3 = \begin{cases} \sqrt{4-2\beta(3-\beta)} - 2(1-\beta), & \text{if } \beta \le \frac{16}{17} \\ \tilde{\rho}_2^c = \frac{\beta\left(12-10\beta-\sqrt{2\beta(2-\beta)}\right)}{4(4-3\beta)}, & \text{otherwise} \end{cases}$$

it is clear that the manufacturer is better off if $\tilde{\rho}_3 < r < \tilde{\rho}_2$.

Next, we consider the retailer. The retailer's profit, $\pi_r^* = \frac{(2(1-\beta)+r)^2}{32(2-\beta)(1-\beta)}$, is also increasing in r in the piracy region $(r < \tilde{\rho}_1)$. Therefore, as before, equating this profit to the benchmark profit of $\pi_{r0} = \frac{1}{16}$ and solving for r, we find that the retailer would also be better off if $r > \sqrt{4-2\beta(3-\beta)} - 2(1-\beta)$ and $\beta \le \frac{16}{17}$. In the threat region $(\tilde{\rho}_1 \le r < \tilde{\rho}_2)$, the retailer's profit, $\pi_r = \frac{2(1-\beta)(\beta-r)^2}{\beta^2(2-\beta)}$, is decreasing in r. This profit is greater than or equal to $\pi_{r0} = \frac{1}{16}$ if and only if $r < \frac{\beta\left((1-\beta)\left(8-2\sqrt{\frac{2(1-\beta)}{2-\beta}}\right)-\beta\sqrt{\frac{2(1-\beta)}{2-\beta}}\right)}{8(1-\beta)}$ and $\beta \le \frac{16}{17}$. We define

$$\tilde{\rho}_4 = \begin{cases} \frac{\beta\left((1-\beta)\left(8-2\sqrt{\frac{2(1-\beta)}{2-\beta}}\right)-\beta\sqrt{\frac{2(1-\beta)}{2-\beta}}\right)}{8(1-\beta)}, & \text{if } \beta \le \frac{16}{17} \\ \tilde{\rho}_2^c = \frac{\beta\left(12-10\beta-\sqrt{2\beta(2-\beta)}\right)}{4(4-3\beta)}, & \text{otherwise} \end{cases}$$

Clearly then, the retailer is better off in the presence of piracy or its threat if $\tilde{\rho}_3 < r < \tilde{\rho}_4$.

Subscription Services and Product Bundling

Assuming that the consumers' valuation for the bundle still follows a uniform distribution over [0,1], and the same degradation factor, β, for both types of pirated content, it is easy to verify that the legal demand is still given by $q(p)$ in (1). The retailer chooses p in order to maximize its profit $\pi_r(p) = (p - w_1 - w_2)\, q(p)$.

If $q(p) = 1 - \frac{p-r}{1-\beta}$, then $\pi_r(p) = (p - w_1 - w_2)\left(1 - \frac{p-r}{1-\beta}\right)$, and by taking the derivative with respect to p, we obtain

$$\frac{\partial \pi_r}{\partial p} = 1 - \frac{2p-r-(w_1+w_2)}{1-\beta} \tag{A6}$$

Since $\frac{\partial^2 \pi_r}{\partial p^2} = -\frac{2}{1-\beta} < 0$, the first-order condition results in $p^*(w\_1, w_2) = \frac{1-\beta+r+w_1+w_2}{2}$, which, according to (1), must be greater than $\frac{r}{\beta}$, or $w_1 + w_2 > \frac{2r}{\beta} - (1-\beta+r)$, for this solution to be valid.

If, on the other hand, $q(p) = 1 - p$, then $\pi_r(p) = (p - w_1 - w_2)(1-p)$, so we get

$$\frac{\partial \pi_r}{\partial p} = 1 - 2p + w_1 + w_2 \tag{A7}$$

Since the second-order condition is trivially satisfied, we can set (A7) to zero to obtain $p^*(w) = \frac{1+w_1+w_2}{2}$, which must be smaller than $\frac{r}{\beta}$, or $w_1 + w_2 < \frac{2r}{\beta} - 1$, for this solution to be valid.

Now, for moderate values of $(w_1 + w_2)$, that is, if $\frac{2r}{\beta} - 1 \le w_1 + w_2 \le \frac{2r}{\beta} - (1-\beta+r)$, $\frac{\partial \pi_r}{\partial p}$ in (A6) is negative, whereas that in (A7) is positive. Naturally, the optimal p is simply $\frac{r}{\beta}$. Taken together, the optimal retail price, $p^*(w_1, w_2)$, can be expressed as

$$p^*(w_1, w_2) = \begin{cases} \frac{1-\beta+r+w_1+w_2}{2}, & \text{if } w_1 + w_2 > \frac{2r}{\beta} - (1-\beta+r) \\ \frac{r}{\beta}, & \text{if } \frac{2r}{\beta} - 1 \le w_1 + w_2 \le \frac{2r}{\beta} - (1-\beta+r) \\ \frac{1+w_1+w_2}{2}, & \text{otherwise} \end{cases} \tag{A8}$$

Now consider the move from manufacturer 1. It anticipates this reaction from the retailer and, given the other manufacturer's wholesale price, w_2, sets its own optimal wholesale price $w_1(w_2)$ to maximize $\pi_{m_1}(w_1, w_2) = w_1\, q(p^*(w_1, w_2))$. As before, we have three cases to consider: (i) $w_1 + w_2 > \frac{2r}{\beta} - (1-\beta+r)$, (ii) $\frac{2r}{\beta} - 1 \le w_1 + w_2 \le \frac{2r}{\beta} - (1-\beta+r)$, and (iii) $w_1 + w_2 < \frac{2r}{\beta} - 1$. For case (i), manufacturer 1 gets a profit of

$$\pi_{m_1} = \frac{w_1(1-\beta+r-(w_1+w_2))}{2(1-\beta)}$$

Since $\frac{\partial^2 \pi_{m_1}}{\partial w_1^2} = -\frac{1}{1-\beta} < 0$, solving the first order condition, $\frac{\partial \pi_{m_1}}{\partial w_1} = \frac{1-\beta+r-2w_1-w_2}{2\,(1-\beta)} = 0$, we get the optimal response function: $w_1^1(w_2) = \frac{1-\beta+r-w_2}{2}$. Similar logic applied to manufacturer 2 gives us its response function as: $w_2^1(w_1) = \frac{1-\beta+r-w_1}{2}$. Simultaneously solving the two response functions, we obtain $w_1^{1*} = w_2^{1*} = \frac{1-\beta+r}{3}$. For this equilibrium to be valid, $(w_1^{1*} + w_2^{1*})$ must be greater than $\frac{2r}{\beta} - (1-\beta+r)$, which is equivalent to $r \le \frac{5\beta(1-\beta)}{6-5\beta} = \tilde{\rho}_1$.

For case (ii), $p^* = \frac{r}{\beta}$. The manufacturers, unwilling to leave money on the table, always choose the highest value from the range $\frac{2r}{\beta} - 1 \le w_1 + w_2 \le \frac{2r}{\beta} - (1-\beta+r)$, resulting in response functions: $w_1^2(w_2) = \frac{2r}{\beta} - (1-\beta+r) - w_2$ and $w_2^2(w_1) = \frac{2r}{\beta} - (1-\beta+r) - w_1$. Once again, simultaneously solving the two response functions, we get $w_1^{2*} = w_2^{2*} = \frac{r}{\beta} - \frac{1-\beta+r}{2}$. To determine the validity of this solution, we note that it must be incentive compatible in the sense that a manufacturer must not have the incentive to deviate to case (i) if the other manufacturer is in case (ii). However, it turns out that

$$\pi_{m_1}\big|_{w_1 = w_1^1(w_2^{2*}), w_2 = w_2^{2*}} - \pi_{m_1}\big|_{w_1 = w_1^2(w_2^{2*}), w_2 = w_2^{2*}} = \frac{\left(5\beta(1-\beta) - r(6-5\beta)\right)^2}{32\beta^2(1-\beta)} \ge 0$$

This is expected; after all, the interior response for a manufacturer should always be better than the boundary response, meaning that case (i) dominates case (ii). However, as we have shown above, case (i) is a valid equilibrium only if $r < \tilde{\rho}_1$. Therefore, for all $r < \tilde{\rho}_1$, the manufacturer would have an incentive to switch from case (ii) to case (i), so case (ii) cannot be a valid equilibrium there. In contrast, if $r \ge \tilde{\rho}_1$, case (i) is not valid, so case (ii) can be a valid equilibrium there.

Finally, in case (iii), $p^*(w_1, w_2) = \frac{1+w_1+w_2}{2}$, and manufacturer 1 gets a profit of $\pi_{m_1} = \frac{w_1(1-(w_1+w_2))}{2}$, which is convex in w_1 and can be easily maximized using the first order condition. The resulting response functions are $w_1^3(w_2) = \frac{1-w_2}{2}$ and $w_2^3(w_1) = \frac{1-w_1}{2}$, implying $w_1^{3*} = w_2^{3*} =$

$\frac{1}{3}$. For this solution to be valid, we must have $w_1^{3*} + w_2^{3*} < \frac{2r}{\beta} - 1$, that is, $r \geq \frac{5\beta}{6} = \tilde{\rho}_5$. Comparing manufacturers' profits in cases (ii) and (iii), we find that case (ii) with prevail over case (iii) if $r < \tilde{\rho}_2 = \frac{\beta(9-6\beta+\sqrt{1+4\beta})}{6(2-\beta)}$. It is easy to verify that $\tilde{\rho}_2 > \tilde{\rho}_5$, making the overall solution spanning the three cases complete.

With the closed-form solution for w_1^* and w_2^*, we can derive p^* from (A8). Therefore, the equilibrium solution is given by

$$w_1^* = w_2^* = \begin{cases} \frac{1-\beta+r}{3}, & \text{if } r < \tilde{\rho}_1 \\ \frac{r}{\beta} - \frac{1-\beta+r}{2}, & \text{if } \tilde{\rho}_1 \leq r < \tilde{\rho}_2, \\ w_0 = \frac{1}{3}, & \text{otherwise} \end{cases} \quad \text{and} \quad p^* = \begin{cases} \frac{5(1-\beta+r)}{6}, & \text{if } r < \tilde{\rho}_1 \\ \frac{r}{\beta}, & \text{if } \tilde{\rho}_1 \leq r < \tilde{\rho}_2 \\ p_0 = \frac{5}{6}, & \text{otherwise} \end{cases}$$

where, as stated earlier, $\tilde{\rho}_1 = \frac{5\beta(1-\beta)}{6-5\beta}$ and $\tilde{\rho}_2 = \frac{\beta(9-6\beta+\sqrt{1+4\beta})}{6(2-\beta)}$. From these, we can now obtain the profits for the manufacturers and the retailer as

$$\pi_{m_1}^* = \pi_{m_2}^* = \begin{cases} \frac{(1-\beta+r)^2}{18(1-\beta)}, & \text{if } r < \tilde{\rho}_1 \\ \frac{(\beta-r)(r-(1-\beta)(\beta-r))}{2\beta^2}, & \text{if } \tilde{\rho}_1 \leq r < \tilde{\rho}_2 \\ \pi_{m0} = \frac{1}{18}, & \text{otherwise} \end{cases} \quad \text{and} \quad \pi_r^* = \begin{cases} \frac{(1-\beta+r)^2}{36(1-\beta)}, & \text{if } r < \tilde{\rho}_1 \\ \frac{(1-\beta)(\beta-r)^2}{\beta^2}, & \text{if } \tilde{\rho}_1 \leq r < \tilde{\rho}_2 \\ \pi_{r0} = \frac{1}{36}, & \text{otherwise} \end{cases}$$

When these profits are compared to their benchmark values, we can obtain the win-win window similar to the one in Theorem 1. First, since in the piracy region $(r < \tilde{\rho}_1)$, the manufacturers' profits, $\pi_{m_1}^* = \pi_{m_2}^* = \frac{(1-\beta+r)^2}{18(1-\beta)}$, are increasing in r, equating these profits to the benchmark profits of $\pi_{m0} = \frac{1}{18}$ and solving for r, we find $r = \sqrt{1-\beta} - (1-\beta)$; of course, for it to be a valid root this r must abide by the restriction $r < \tilde{\rho}_1$, which is equivalent to $\beta < \frac{24}{25}$. Next, in the threat region $(\tilde{\rho}_1 \leq r < \tilde{\rho}_2)$, the manufacturers' profits, $\pi_{m_1}^* = \pi_{m_2}^* = \frac{(\beta-r)(r-(1-\beta)(\beta-r))}{2\beta^2}$, can never be less than π_{m0}. In other words, for all $\beta \leq \frac{24}{25}$, a necessary and sufficient condition for the manufacturer to be better off is $\sqrt{1-\beta} - (1-\beta) < r < \tilde{\rho}_2$.

The case of $\beta > \frac{24}{25}$ is somewhat different. Here, the threat region takes over at a lower r; the profit function for the threat region meets the benchmark profit, $\pi_{m0} = \frac{1}{18}$, two times, first at point $\tilde{\rho}_2^c$ and then again at $\tilde{\rho}_2$, where $\tilde{\rho}_2^c$ is the root conjugate to $\tilde{\rho}_2$ and is given by

$$\tilde{\rho}_2^c = \frac{\beta\left(9-6\beta-\sqrt{1+4\beta}\right)}{6(2-\beta)}$$

Therefore, for all $\beta > \frac{24}{25}$, the manufacturer would be better off if and only if $\tilde{\rho}_2^c < r < \tilde{\rho}_2$. Defining

$$\tilde{\rho}_3 = \begin{cases} \sqrt{1-\beta} - (1-\beta), & \text{if } \beta \leq \frac{24}{25} \\ \tilde{\rho}_2^c = \frac{\beta\left(9-6\beta-\sqrt{1+4\beta}\right)}{6(2-\beta)}, & \text{otherwise} \end{cases}$$

it is clear that the manufacturers are better off if $\tilde{\rho}_3 < r < \tilde{\rho}_2$.

Next, we consider the retailer. The retailer's profit, $\pi_{r^*} = \frac{(1-\beta+r)^2}{36(1-\beta)}$, is also increasing in r in the piracy region $(r < \tilde{\rho}_1)$. Therefore, as before, equating this profit to the benchmark profit of $\pi_{r0} = \frac{1}{36}$ and solving for r, we find that the retailer would also be better off if $r > \sqrt{1-\beta} - (1-\beta)$ and $\beta \leq \frac{24}{25}$. In the threat region $(\tilde{\rho}_1 \leq r < \tilde{\rho}_2)$, the retailer's profit, $\pi_r^* = \frac{(1-\beta)(\beta-r)^2}{\beta^2}$, is decreasing in r. This profit is greater than or equal to $\pi_{r0} = \frac{1}{36}$ if and only if $r < \beta\left(1 - \frac{1}{6\sqrt{1-\beta}}\right)$ and $\beta \leq \frac{24}{25}$. We define

$$\tilde{\rho}_4 = \begin{cases} \beta\left(1 - \frac{1}{6\sqrt{1-\beta}}\right), & \text{if } \beta \leq \frac{24}{25} \\ \tilde{\rho}_2^c = \frac{\beta\left(9-6\beta-\sqrt{1+4\beta}\right)}{6(2-\beta)}, & \text{otherwise} \end{cases}$$

Clearly then, the retailer is better off in the presence of piracy or its threat if $\tilde{\rho}_3 < r < \tilde{\rho}_4$.

Piracy Cost Recouped by the Legal Channel

Recall that the demands for the legal and illegal versions at a given retail price p are exactly as those in our original model in (1). However, in this extension, the manufacturer and retailer also make an additional $\mu\lambda r$ and $(1-\mu)\lambda r$, respectively, for every unit of illegal product sold. Using (1), the resulting profit functions for the manufacturer and the retailer can then be written as

$$\pi_m = \begin{cases} w\left(1-\frac{p-r}{1-\beta}\right)+\mu\lambda r\left(\frac{p-r}{1-\beta}-\frac{r}{\beta}\right), & \text{if } p>\frac{r}{\beta} \\ w(1-p), & \text{otherwise} \end{cases} \tag{A9}$$

$$\pi_r = \begin{cases} (p-w)\left(1-\frac{p-r}{1-\beta}\right)+(1-\mu)\lambda r\left(\frac{p-r}{1-\beta}-\frac{r}{\beta}\right), & \text{if } p>\frac{r}{\beta} \\ (p-w)(1-p), & \text{otherwise} \end{cases} \tag{A10}$$

As a result, the optimal prices differ from those in the original model. The optimal retail price for a given w, $p^*(w)$, can now be found by maximizing π_r in (A10). Repeating exactly the same method we used for deriving Lemma 1, we can easily derive an analogous expression for the optimal p in this extended setup

$$p^*(w) = \begin{cases} \frac{1-\beta+r(1+\lambda(1-\mu))+w}{2}, & \text{if } w>\frac{2r}{\beta}-r(1+\lambda(1-\mu))-(1-\beta) \\ \frac{r}{\beta}, & \text{if } \frac{2r}{\beta}-1\le w\le\frac{2r}{\beta}-r(1+\lambda(1-\mu))-(1-\beta) \\ \frac{1+w}{2}, & \text{otherwise} \end{cases}$$

Note that, when $\lambda=0$, this $p^*(w)$ coincides with that given in Lemma 1. We are now ready to characterize the new ρ_i thresholds, $i\in\{1,2,3,4,5\}$; to avoid confusion with our original notation, we denote the new ones as $\tilde{\rho}_i$ here.

Once the retailer's response, $p^*(w)$, is known, the manufacturer's problem is to maximize $w(1-p^*(w))$. It is clear from the expression of $p^*(w)$ above that we have three cases to consider: (i) $w>\frac{2r}{\beta}-r(1+\lambda(1-\mu))-(1-\beta)$, (ii) $\frac{2r}{\beta}-1\le w\le\frac{2r}{\beta}-r(1+\lambda(1-\mu))-(1-\beta)$, and (iii) $w<\frac{2r}{\beta}-1$.

For case (i), $p^*(w)=\frac{1-\beta+r(1+\lambda(1-\mu))+w}{2}$, and the first order condition with respect to w results in $w^*=\frac{1-\beta+r(1+\lambda(2\mu-1))}{2}$. This solution must be greater than $\frac{2r}{\beta}-r(1+\lambda(1-\mu))-(1-\beta)$, which leads to $r<\tilde{\rho}_1=\frac{3\beta(1-\beta)}{4-3\beta(1+\lambda)}$.

For case (ii), $p^*=\frac{r}{\beta}$. The manufacturer, unwilling to leave money on the table, always chooses the highest value from the range $\left[\frac{2r}{\beta}-1,\frac{2r}{\beta}-r(1+\lambda(1-\mu))-(1-\beta)\right]$, resulting in $w^*=\frac{2r}{\beta}-r(1+\lambda(1-\mu))-(1-\beta)$. This equilibrium is valid across all $r\le\beta$. As was the case in our original model, $r>\beta$ still falls under case (iii), in which no consumer considers the pirated product as an option.

Finally, in case (iii), $p^*(w)=\frac{1+w}{2}$, which leads to $w^*=\frac{1}{2}$. This w^* must be less than $\frac{2r}{\beta}-1$, implying that $r\ge\frac{3\beta}{4}=\tilde{\rho}_5$ must hold for case (iii) to occur.

Now, case (ii) is the only valid equilibrium if $\tilde{\rho}_1\le r\le\tilde{\rho}_5$. On one hand, if $r<\tilde{\rho}_1$, both cases (i) and (ii) are valid. However, the optimal profit from an interior solution ought to be higher, which immediately implies that case (i) is the equilibrium outcome for $r<\tilde{\rho}_1$. If, on the other hand, $r>\tilde{\rho}_5$, both cases (ii) and (iii) are possible, and we must compare the manufacturer's profit in these two cases to determine the equilibrium. We can obtain the optimal profits for these two cases using the w^* for the respective cases. The optimal profit for case (ii) is $\frac{(\beta-r)(r(2-\beta(1+\lambda(1-\mu)))-\beta(1-\beta))}{\beta^2}$. The optimal profit for case (iii) is simply $\frac{1}{8}$. Accordingly, $\tilde{\rho}_2$, the boundary between the limit and benchmark regions, is given by

$$\tilde{\rho}_2=\beta\left(1-\frac{2(1-\beta\lambda(1-\mu))-\sqrt{2\beta(1-\lambda(1-\mu)(3-2\beta\lambda(1-\mu)))}}{4\left(2-\beta\left(1+\lambda(1-\mu)\right)\right)}\right)$$

Now, let us turn to the win-win region. Unlike in our original model, it is no longer true that the manufacturer wins whenever the retailer wins. This is because, when μ is small and consequently $(1-\mu)$ is large, the retailer may win while the manufacturer loses. Therefore, to find $\tilde{\rho}_3$, we must first find the thresholds for the manufacturer and retailer separately. Once we know the thresholds above which the manufacturer and retailer are better off, we can take their maximum to determine $\tilde{\rho}_3$. The threshold for the manufacturer, $\tilde{\rho}_{3m}$, is obtained by equating the manufacturer's profit in the piracy region with that in the benchmark region. The manufacturer's profit in the piracy region is

$$\frac{2\beta r(1-\beta)(1-\lambda(1-4\mu)) + (1-\beta)^2 + r^2(\beta(1-\lambda)^2 - 8\lambda\mu(1-\beta))}{8\beta(1-\beta)}$$

Since the profit in the benchmark region is $\frac{1}{8}$, we get

$$\tilde{\rho}_{3m} = \frac{\beta}{1-\lambda(1-4\mu) + \sqrt{\frac{1-\lambda(2-\lambda(1-8\mu(1-\beta)(1-2\mu)))}{1-\beta}}}$$

Similarly, we can solve the retailer's threshold. Its profit in the piracy region is

$$\frac{2\beta r(1-\beta)(1+\lambda(7-8\mu)) + \beta(1-\beta)^2 + r^2(\beta(1-\lambda)^2 - 16\lambda(1-\beta)(1-\mu))}{16\beta(1-\beta)}$$

The retailer makes $\frac{1}{16}$ in the benchmark region. It immediately follows that

$$\tilde{\rho}_{3r} = \frac{\beta}{1-\lambda(7-8\mu) + \sqrt{\frac{1-\lambda(2-\lambda((7-8\mu)^2-16\beta(1-\mu)(3-4\mu)))}{1-\beta}}}$$

It is easy to verify that $\tilde{\rho}_{3m} > \tilde{\rho}_{3r}$ for $\mu < \frac{2}{3}$, which leads to

$$\tilde{\rho}_3 = \begin{cases} \tilde{\rho}_{3m}, & \text{if } \mu < \frac{2}{3} \\ \tilde{\rho}_{3r}, & \text{otherwise} \end{cases}$$

Now, to solve for $\tilde{\rho}_4$, we need to compare the profit in case (ii) with the benchmark profit in case (iii). We again do this exercise separately for the manufacturer and retailer to obtain $\tilde{\rho}_{4m}$ and $\tilde{\rho}_{4r}$, respectively. The upper bound of the win-win region, $\tilde{\rho}_4$, is then the smaller of these two thresholds. Note that, by definition,

$$\tilde{\rho}_{4m} = \tilde{\rho}_2$$

and $\tilde{\rho}_{4r}$ is the solution of

$$\frac{(\beta - r)(\beta(1-\beta) - r(1-\beta(1+\lambda(1-\mu))))}{\beta^2} = \frac{1}{16}$$

Therefore,

$$\tilde{\rho}_{4r} = \beta\left(1 + \frac{2\beta\lambda(1-\mu) - \sqrt{1-\beta(1+\lambda(1-\mu)(1-4\beta\lambda(1-\mu)))}}{4(1-\beta(1+\lambda(1-\mu)))}\right)$$

Comparing $\tilde{\rho}_{4m}$ with $\tilde{\rho}_{4r}$, we can derive $\tilde{\rho}_4$:

$$\tilde{\rho}_4 = \begin{cases} \tilde{\rho}_{4m}, & \text{if } \lambda(1-\mu) > \frac{1}{3} \\ \tilde{\rho}_{4r}, & \text{otherwise} \end{cases}$$

Finally, as shown in the paper, for the win-win region to exist, both $\tilde{\rho}_3$ and $\tilde{\rho}_4$ must be real and must satisfy $\tilde{\rho}_3 < \tilde{\rho}_4$.

Network Effect

Since we now assume a consumer's valuation to $v(1+\Gamma)$, the demand for the legal product becomes

$$q(p)=\begin{cases}1-\frac{p-r}{(1-\beta)(1+\Gamma)}, & \text{if } p>\frac{r}{\beta}\\ 1-\frac{p}{1+\Gamma}, & \text{otherwise}\end{cases}$$

which can also be rewritten as

$$q(p)=\begin{cases}1-\frac{p-r}{(1-\beta)(1+\Gamma_{\text{piracy}})}, & \text{if } p>\frac{r}{\beta}\\ 1-\frac{p}{1+\Gamma_{\text{threat}}}, & \text{if } p=\frac{r}{\beta}\\ 1-\frac{p}{1+\Gamma_{\text{benchmark}}}, & \text{otherwise}\end{cases} \tag{A11}$$

Let us first consider the piracy region $\left(p>\frac{r}{\beta}\right)$ and the threat region $\left(p=\frac{r}{\beta}\right)$, where the marginal consumer, $\bar{v}$, can be characterized by $\bar{v}=\frac{r}{\beta(1+\Gamma)}$. Since $\Gamma=\gamma(1-\bar{v})$ by definition, in a fulfilled expectations equilibrium, the following must hold:

$$\Gamma=\gamma(1-\bar{v})=\gamma\left(1-\frac{r}{\beta(1+\Gamma)}\right)$$

Solving this, we obtain the equilibrium Γ as follows:

$$\Gamma_{\text{piracy}}=\Gamma_{\text{threat}}=\frac{1}{2}\left(\gamma-1+\sqrt{(\gamma+1)^2-\frac{4r\gamma}{\beta}}\right)$$

Now, let us consider the benchmark region where $p<\frac{r}{\beta}$. Starting with the demand expression in (A11), it is straightforward to show that the equilibrium price set by the retailer is simply $p=\frac{3(1+\Gamma)}{4}$, which means, exactly as in our original model, only a quarter of the market gets covered in equilibrium regardless of the actual value of Γ, implying that $\bar{v}=\frac{3}{4}$. Hence, the equilibrium Γ must be

$$\Gamma_{\text{benchmark}}=\gamma(1-\bar{v})=\frac{\gamma}{4}$$

With the demand so characterized, we can now proceed to solve for the thresholds $\tilde{\rho}_i$ that are analogous to the thresholds ρ_i in Theorem 1, for $i\in\{1,2,3,4,5\}$. Recall that $\Gamma_{\text{piracy}}=\Gamma_{\text{threat}}$; we will henceforth call them both Γ_a for convenience. Likewise, we will use a shorter notation Γ_b to denote $\Gamma_{\text{benchmark}}$. We proceed exactly the same way we solved our original model. Repeating the steps in Lemma 1, we can easily derive the optimal p as

$$p^*(w)=\begin{cases}\frac{(1-\beta)(1+\Gamma_a)+r+w}{2}, & \text{if } w>\frac{2r}{\beta}-r-(1-\beta)(1+\Gamma_a)\\ \frac{r}{\beta}, & \text{if } \frac{2r}{\beta}-1-\Gamma_b\le w\le\frac{2r}{\beta}-r-(1-\beta)(1+\Gamma_a)\\ \frac{1+w+\Gamma_b}{2}, & \text{otherwise}\end{cases}$$

Clearly, $\gamma=0$ implies that $\Gamma_a=\Gamma_b=0$. As a result, when $\gamma=0$, $p^*(w)$ above coincides with that given in Lemma 1.

Once the retailer's response $p^*(w)$ is known, the manufacturer's problem is simply to maximize $w(1-p^*(w))$. It is clear from the expression of $p^*(w)$ above that we have three cases to consider: (i) $w>\frac{2r}{\beta}-r-(1-\beta)(1+\Gamma_a)$, (ii) $\frac{2r}{\beta}-1-\Gamma_b\le w\le\frac{2r}{\beta}-r-(1-\beta)(1+\Gamma_a)$, and (iii) $w<\frac{2r}{\beta}-1-\Gamma_b$.

For case (i), $p^*(w)=\frac{(1-\beta)(1+\Gamma_a)+r+w}{2}$, and the first order condition with respect to w results in $w^*=\frac{(1-\beta)(1+\Gamma_a)+r}{2}$. This solution must be greater than $\frac{2r}{\beta}-r-(1-\beta)(1+\Gamma_a)$, which leads to $r<\tilde{\rho}_1$ where $\tilde{\rho}_1$ is the solution of

$$\frac{(1-\beta)(1+\Gamma_a)+r}{2}=\frac{2r}{\beta}-r-(1-\beta)(1+\Gamma_a)$$

Substituting $\Gamma_a=\frac{1}{2}\left(\gamma-1+\sqrt{(\gamma+1)^2-\frac{4r\gamma}{\beta}}\right)$ and subsequently solving the above, we get

$$\tilde{\rho}_1=\frac{3\beta(1-\beta)(4-3\beta+\gamma)}{(4-3\beta)^2}$$

For case(ii), $p^*=\frac{r}{\beta}$ and, once again, the manufacturer, unwilling to leave money on the table, chooses the highest value from the range $\left[\frac{2r}{\beta}-1-\Gamma_b,\frac{2r}{\beta}-r-(1-\beta)(1+\Gamma_a)\right]$, resulting in $w^*=\frac{2r}{\beta}-r-(1-\beta)(1+\Gamma_a)$. As was the case in our original model, this equilibrium is valid across all $r\leq\beta$ and the case of $r>\beta$ falls under case (iii) where the pirated product is not an option.

Finally, in case (iii), $p^*(w)=\frac{1+w+\Gamma_b}{2}$, which leads to $w^*=\frac{1+\Gamma_b}{2}$. This w^*must be less than $\frac{2r}{\beta}-1-\Gamma_b$, where $\Gamma_b=\frac{\gamma}{4}$, implying that $r\geq\frac{3\beta}{4}\left(1+\frac{\gamma}{4}\right)=\tilde{\rho}_5$ must hold for case (iii) to occur.

Now, case (ii) is the only valid equilibrium if $\tilde{\rho}_1\leq r\leq\tilde{\rho}_5$. On the one hand, if $r<\tilde{\rho}_1$, both cases (i) and (ii) are valid. However, the optimal profit from an interior solution ought to be higher, which immediately implies that case (i) is the equilibrium outcome for $r<\tilde{\rho}_1$. If, on the other hand, $r>\tilde{\rho}_5$, both cases (ii) and (iii) are possible, and we must compare the manufacturer's profit in these two cases to determine the equilibrium. We can obtain the optimal profits for these two cases using the w^* for the respective cases. The optimal profit for case (ii) is $\left(\frac{2r}{\beta}-r-(1-\beta)(1+\Gamma_a)\right)\left(1-\frac{r}{\beta(1+\Gamma_a)}\right)$, where, as before, $\Gamma_a=\frac{1}{2}\left(\gamma-1+\sqrt{(\gamma+1)^2-\frac{4r\gamma}{\beta}}\right)$. The optimal profit for case (iii) is $\frac{1+\Gamma_b}{8}$, where $\Gamma_b=\frac{\gamma}{4}$. Hence, $\tilde{\rho}_2$ can be obtained as the larger of the two positive roots of

$$\left(\frac{2r}{\beta}-r-\frac{1-\beta}{2}\left(\gamma+1+\sqrt{(\gamma+1)^2-\frac{4r\gamma}{\beta}}\right)\right)\left(1-\frac{r}{\frac{\beta}{2}\left(\gamma+1+\sqrt{(\gamma+1)^2-\frac{4r\gamma}{\beta}}\right)}\right)=\frac{1+\frac{\gamma}{4}}{8}$$

A closed-form solution does exist, but the size of the expression precludes it from this appendix.

Now, let us turn to the win-win region. To solve for $\tilde{\rho}_3$, we need to equate the retailer's profit in case (i) with the benchmark profit, that is, the profit in case (iii). The profit in case (i) is $\frac{(r+(1-\beta)(1+\Gamma_a))^2}{16\,(1-\beta)(1+\Gamma_a)}$ while that in case (iii) is $\frac{1+\Gamma_b}{16}$. Hence, $\tilde{\rho}_3$ is the root in $[0,\tilde{\rho}_1]$ of the following:

$$\frac{\left(r+\frac{1-\beta}{2}\left(\gamma+1+\sqrt{(\gamma+1)^2-\frac{4r\gamma}{\beta}}\right)\right)^2}{8\,(1-\beta)\left(\gamma+1+\sqrt{(\gamma+1)^2-\frac{4r\gamma}{\beta}}\right)}=\frac{1+\frac{\gamma}{4}}{16}$$

Again, although a closed form solution exists, it is simply too long and cumbersome to report here.

Finally, to obtain $\tilde{\rho}_4$, we need to equate the retailer's profit in case (ii) with the benchmark profit. The profit in case (ii) is $\frac{(1-\beta)(\beta(1+\Gamma_a)-r)^2}{\beta^2(1+\Gamma_a)}$ and that in case (iii) is as mentioned above. Thus, $\tilde{\rho}_4$ is the root in $[\tilde{\rho}_1,\tilde{\rho}_2]$ of the following:

$$\frac{(1-\beta)\left(\frac{\beta}{2}\left(\gamma+1+\sqrt{(\gamma+1)^2-\frac{4r\gamma}{\beta}}\right)-r\right)^2}{\frac{\beta^2}{2}\left(\gamma+1+\sqrt{(\gamma+1)^2-\frac{4r\gamma}{\beta}}\right)}=\frac{1+\frac{\gamma}{4}}{16}$$

When $\gamma\to 0$, all the $\tilde{\rho}_i$ thresholds, $i\in\{1,2,3,4,5\}$, nicely converge to ρ_i in our original model.

Downstream Competition

Given the straightforward setting of a horizontal market, the fraction of consumers who prefers retailer A B is simply $\left(\frac{1}{2}+\frac{p_B-p_A}{2\delta}\right)$. Likewise, the remaining $\left(\frac{1}{2}+\frac{p_A-p_B}{2\delta}\right)$ fraction prefers B to A. It is easy to see that, when δ becomes large, the two markets separate; essentially, the weakened competition empowers the individual retailers as local monopolies, and our earlier results apply.

Now, irrespective of the value of δ, each consumer has to make a choice among: (i) buying the legal product from his preferred retailer, (ii) using an illegal copy, and (iii) not using the product at all. We consider this choice to be independent of the consumer's preference for a retailer. In other words, we continue to assume that this choice is still governed by the IR and IC constraints discussed in the consumer behavior section in the paper. Accordingly, the legal demand for retailer A can now be expressed as

$$q_A(p_A,p_B)=\begin{cases}\left(\frac{1}{2}+\frac{p_B-p_A}{2\delta}\right)\left(1-\frac{p_A-r}{1-\beta}\right), & \text{if } p_A>\frac{r}{\beta}\\ \left(\frac{1}{2}+\frac{p_B-p_A}{2\delta}\right)(1-p_A), & \text{otherwise}\end{cases} \tag{A12}$$

Retailer A maximizes $(p_A-w)q_A(p_A,p_B)$, and retailer B, $(p_B-w)q_B(p_B,p_A)$. As before, three regions emerge and a retailer prefers to employ the limit price only when w is moderate. Specifically, retailer A chooses $p_A=\frac{r}{\beta}$ if $w_{AL}\le w\le w_{AH}$, where

$$w_{AL}=\frac{3r^2+\beta^2(p_B+\delta)-2\beta r(1+p_B+\delta)}{\beta(2r-\beta(1+p_B+\delta))} \quad \text{and}$$
$$w_{AH}=\frac{r^2(3-2\beta)+\beta^2(1-\beta)(p_B+\delta)-\beta r(p_B(2-\beta)+2(1+\delta)-\beta(2+\delta))}{\beta(r(2-\beta)-\beta(1+p_B-\beta+\delta))}$$

A similar range exists for retailer B as well. Therefore, a symmetric equilibrium with $p_A=p_B=\frac{r}{\beta}$ is possible only if w is between the following two limits:

$$w_L=w_{AL}\big|_{p_B=\frac{r}{\beta}}=\frac{\beta r(1+2\delta)-r^2-\beta^2\delta}{\beta(\beta(1+\delta)-r)} \quad \text{and}$$
$$w_H=w_{AH}\big|_{p_B=\frac{r}{\beta}}=\frac{r(1-\beta)(\beta-r)+\beta\delta(r(2-\beta)-\beta(1-\beta))}{\beta(\beta(1+\delta)-r-\beta(\beta-r))}$$

Note that, if the manufacturer sets $w>w_H$, the only possible symmetric equilibrium is the one in which both retailers name a price above $\frac{r}{\beta}$. Retailer A's optimal price in this equilibrium is obtained from

$$\frac{\partial}{\partial p_A}\left((p_A-w)\left(1-\frac{p_A-r}{1-\beta}\right)\left(\frac{1}{2}+\frac{p_B-p_A}{2\delta}\right)\right)\Bigg|_{p_B=p_A}=0$$

leading to $p_A=p_B=\frac{1-\beta+r+w+2\delta-\sqrt{(1-\beta+r-w)^2+4\delta^2}}{2}$. On the other hand, if the manufacturer chooses a wholesales price below w_L, the symmetric equilibrium of interest would be the one in which $p_A=p_B<\frac{r}{\beta}$. Retailer A's first order condition in this case is

$$\frac{\partial}{\partial p_A}\left((p_A-w)(1-p_A)\left(\frac{1}{2}+\frac{p_B-p_A}{2\delta}\right)\right)\Bigg|_{p_B=p_A}=0$$

which leads to $p_A=p_B=\frac{1+w+2\delta-\sqrt{(1-w)^2+4\delta^2}}{2}$.

Putting all of the above elements together, in a symmetric equilibrium, the optimal retail price for a given w, $p^*(w)=p_A^*(w)=p_B^*(w)$, must satisfy

$$p^*(w)=\begin{cases}\frac{1-\beta+r+w+2\delta-\sqrt{(1-\beta+r-w)^2+4\delta^2}}{2}, & \text{if } w>w_H\\ \frac{r}{\beta}, & \text{if } w_L\le w\le w_H\\ \frac{1+w+2\delta-\sqrt{(1-w)^2+4\delta^2}}{2}, & \text{otherwise}\end{cases}$$

Since this expression is similar to the one in Lemma 1 in the paper, the rest of the derivation of the equilibrium is not conceptually any harder. In particular, given this $p^*(w)$, the manufacturer chooses w^*, the optimal w that maximizes its profit, $\pi_m(w) = 2w \times (q^*(p^*(w))$, where $q^*(p^*(w))$ is obtained by setting $p_A = p_B = p^*(w)$ in~(A12). In the piracy region $(w > w_H)$, as well as in the benchmark region $(w < w_L)$, the manufacturer's profit is concave in the region of interest, and a unique w^* can be found from the first order condition, although the size of its expression in Mathematica precludes reporting it in this appendix. Finally, in the threat region $(w_L \leq w \leq w_H)$, the manufacturer prefers w_H to any other $w \in [w_L, w_H]$ while inducing the retailers to choose $p^*(w) = \frac{r}{\beta}$ in equilibrium.

By a chain of backward substitutions of this w^*, we can find the optimal retail price, $p^*(w^*)$ and the optimal demand $q^*(p^*(w^*))$. Therefore, the equilibrium profits of the manufacturer and retailers can also be found. Fortunately, unique closed form expressions still exist; it is just that they are simply too large to report here. Instead, we illustrate their behavior in Figure A4, where these profits are plotted as functions r and β. Once again, even in this case, the red-blue humps over the benchmark level, reminiscent of a win-win window, are unmistakably visible. Therefore, to establish the existence of a win-win window, all that remains is to show that there is some overlap between the two humps in the two profit plots. In particular, let $(\tilde{\rho}_{3m}, \tilde{\rho}_2)$ be the manufacturer's winning window and $(\tilde{\rho}_{3r}, \tilde{\rho}_4)$, the retailers'. These windows can be analytically obtained and plotted, as shown in Figure 12. The overlap between them is clearly visible in the figure.

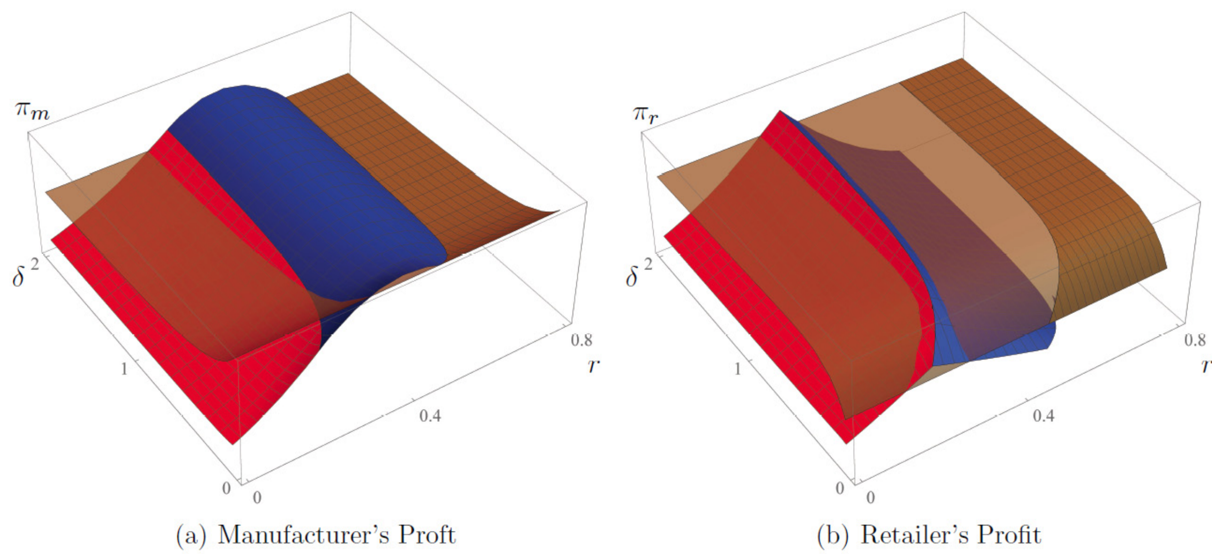


Figure A4. Profit as a Function of r and δ; $\beta = 0.75$

Appendix B

Proofs

Proof of Lemma 1

If $q(p) = 1 - \frac{p-r}{1-\beta}$, then $\pi_r(p) = (p-w)\left(1 - \frac{p-r}{1-\beta}\right)$, implying

$$\frac{\partial \pi_r}{\partial p} = 1 - \frac{2p-r-w}{1-\beta} \tag{B1}$$

Since $\frac{\partial^2 \pi_r}{\partial p^2} = -\frac{2}{1-\beta} < 0$, the first order condition results in $p^*(w) = \frac{1}{2}(1-\beta+r+w)$, which according to (1), must be greater than $\frac{r}{\beta}$, or $w > \frac{2r}{\beta} - (1-\beta+r)$, for this solution to be valid.

If, on the other hand, $q(p) = 1-p$, then $\pi_r(p) = (p-w)(1-p)$, resulting in

$$\frac{\partial \pi_r}{\partial p} = 1 - 2p + w \tag{B2}$$

Furthermore, since $\frac{\partial^2 \pi_r}{\partial p^2} = -2 < 0$, $\frac{\partial \pi_r}{\partial p} = 0$ results in $p^*(w) = \frac{1+w}{2}$, which must be smaller than $\frac{r}{\beta}$, or $w < \frac{2r}{\beta} - 1$, for this solution to be valid.

Now, for moderate values of w, that is, if $\frac{2r}{\beta} - 1 \le w \le \frac{2r}{\beta} - (1-\beta+r)$, $\frac{\partial \pi_r}{\partial p}$ given by (B1) is negative whereas that given by (B2) is positive. Naturally, the optimal p is simply $\frac{r}{\beta}$. ■

Proof of Proposition 1

From Lemma 1, it is evident that we have three cases to consider: (i) w $> \frac{2r}{\beta} - (1-\beta+r)$, (ii) $\frac{2r}{\beta} - 1 \le w \le \frac{2r}{\beta} - (1-\beta+r)$ and (iii) $w < \frac{2r}{\beta} - 1$.

For case (i), we substitute $p^*(w) = \frac{1}{2}(1-\beta+r+w)$ into (1) to obtain the manufacturer's profit

$$\pi_m = \frac{w(1-\beta+r-w)}{2(1-\beta)} \tag{B3}$$

Since $\frac{\partial^2 \pi_m}{\partial w^2} = -\frac{1}{1-\beta} < 0$, the first order condition, $\frac{\partial \pi_m}{\partial m} = \frac{1-\beta+r-2w}{2(1-\beta)} = 0$, results in $w^* = \frac{1-\beta+r}{2}$, which, according to Lemma 1, must be greater than $\frac{2r}{\beta} - (1-\beta+r)$, or $r < \frac{3\beta(1-\beta)}{4-3\beta} = \rho_1$, for this equilibrium to be valid.

For case (ii), $p^* = \frac{r}{\beta}$. The manufacturer, unwilling to leave money on the table, always chooses the highest value from the range $\frac{2r}{\beta} - 1 \le w \le \frac{2r}{\beta} - (1-\beta+r)$, resulting in $w^* = \frac{2r}{\beta} - (1-\beta+r)$. This equilibrium is valid across all $r \le \beta$. Point to note Point to note here is that, if $r > \beta$, then r is also greater than ρ_2, which can be shown to be less than β. Therefore, $r > \beta$ falls under case (iii), the benchmark region, which we discuss next. Viewed differently, if $r > \beta$, then $p^* = \frac{r}{\beta} > 1$, and no consumer would buy the product. Therefore, $r > \beta$ cannot happen in case (ii).

Finally, in case (iii), $p^*(w) = \frac{1+w}{2}$, and the manufacturer's profit is

$$\pi_m = \frac{w(1-w)}{2} \tag{B4}$$

implying $w^* = \frac{1}{2}$. According to Lemma 1, this w^* must be less than $\frac{2r}{\beta} - 1$, or $r > \frac{3\beta}{4} = \rho_5$.

Since $\rho_1 < \rho_5$, (ii) is the only valid equilibrium if $\rho_1 \leq r \leq \rho_5$, and $w^* = \frac{2r}{\beta} - (1 - \beta + r)$. If $r < \rho_1$, the manufacturer can either set $w = \frac{1-\beta+r}{2}$, or set $w = \frac{2r}{\beta} - (1 - \beta + r)$. If the manufacturer chooses $w = \frac{1-\beta+r}{2}$, its profit is $\frac{(1-\beta+r)^2}{8(1-\beta)}$ from (B3). On the other hand, if it chooses $w = \frac{2r}{\beta} - (1 - \beta + r)$, its profit becomes $w\left(1 - \frac{r}{\beta}\right) = \frac{(\beta-r)(r-(1-\beta)(\beta-r))}{\beta^2}$. Between the two choices, the manufacturer chooses the one that yields a higher profit. It is easy to verify that, at $r = \rho_1$, both options yield the same profit, and for $r < \rho_1$, the first option is always better. Thus, if $r < \rho_1$, (i) is the equilibrium outcome and $w^* = \frac{1-\beta+r}{2}$.

If $r > \rho_5$, the manufacturer can either set $w = \frac{1}{2}$, or set $w = \frac{2r}{\beta} - (1 - \beta + r)$. If $w = \frac{1}{2}$, the manufacturer's profit is $\frac{1}{8}$ from (B4), and, if $w = \frac{2r}{\beta} - (1 - \beta + r)$, the profit becomes, as before, $\frac{(\beta-r)(r-(1-\beta)(\beta-r))}{\beta^2}$. Comparing these two profits, it is easy to verify that the manufacturer would choose the first option if $r \geq \frac{\beta(6-4\beta+\sqrt{2\beta})}{4(2-\beta)} = \rho_2$. Since $\rho_2 > \rho_5$ holds trivially, (iii) is the equilibrium outcome with $w^* = \frac{1}{2}$ if $r \geq \rho_2$. By the same logic, for $\rho_5 \leq r < \rho_2$, case (ii) is the equilibrium. It should now be clear from the preceding discussion that case (ii) is the equilibrium for the entire range $\rho_1 \leq r < \rho_2$.

With the closed-form solution for w^*, we can derive p^* from Lemma 1. ■

Proof of Proposition 2

Using p^* and w^* from Proposition 1, we can find the equilibrium profits for the manufacturer and retailer as $\pi_m^* = w^* \, q(p^*)$ and $\pi_r^* = (p^* - w^*) \, q(p^*)$, respectively. ■

Proof of Theorem 1

First, since in the piracy region $r < \rho_1$, the manufacturer's profit, $\pi_m^* = \frac{(1-\beta+r)^2}{8(1-\beta)}$, is increasing in r, equating this profit to the benchmark profit of $\pi_{m0} = \frac{1}{8}$ and solving for r, we find $r = \sqrt{1-\beta} - (1-\beta)$; of course, for it to be a valid root this r must abide by the restriction $r < \rho_1$, which is equivalent to $\beta < \frac{8}{9}$. Next, in the threat region ($\rho_1 \leq r \leq \rho_2$), the manufacturer's profit, $\pi_m^* = \frac{(\beta-r)(r-(1-\beta)(\beta-r))}{\beta^2}$, can never be less than π_{m0}. In other words, for all $\beta < \frac{8}{9}$, a necessary and sufficient for the manufacturer to be better off is $\sqrt{1-\beta} - (1-\beta) < r < \rho_2$.

The case of $\beta > \frac{8}{9}$ is somewhat different. Here, the threat region takes over at a lower r; the profit function for the threat region meets the benchmark profit, $\pi_{m0} = \frac{1}{8}$, two times, first at point ρ_2^c and then again at ρ_2, where ρ_2^c is the root conjugate to ρ_2 and is given by

$$\rho_2^c = \frac{\beta(6-4\beta-\sqrt{2\beta})}{4(2-\beta)}$$

Therefore, for all $\beta > \frac{8}{9}$, the manufacturer would be better off if and only if $\rho_2^c < r < \rho_2$. Define

$$\rho_3 = \begin{cases} \sqrt{1-\beta} - (1-\beta), & \text{if } \beta \leq \frac{8}{9} \\ \rho_2^c = \frac{\beta(6-4\beta-\sqrt{2\beta})}{4(2-\beta)}, & \text{otherwise} \end{cases}$$

It is then immediate that the manufacturer is better off if $\rho_3 < r < \rho_2$.

Next, we consider the retailer. The retailer's profit, $\pi_r^* = \frac{(1-\beta+r)^2}{16(1-\beta)}$, is also increasing in r in the piracy region ($r < \rho_1$). Therefore, as before, equating this profit to the benchmark profit of $\pi_{r0} = \frac{1}{16}$ and solving for r, we find that the retailer would also be better off if $r > \sqrt{1-\beta} -$

$(1-\beta)$ and $\beta \leq \frac{8}{9}$. In the threat region ($\rho_1 \leq r < \rho_2$), the retailer's profit, $\pi_r^* = \frac{(1-\beta)(\beta-r)^2}{\beta^2}$, is decreasing in r. This profit is greater than or equal to π_{r0} if and only if $r < \beta\left(1 - \frac{1}{4\sqrt{1-\beta}}\right)$ and $\beta \leq \frac{8}{9}$. We define

$$\rho_4 = \begin{cases} \beta\left(1 - \frac{1}{4\sqrt{1-\beta}}\right), & \text{if } \beta \leq \frac{8}{9} \\ \rho_2^c = \frac{\beta(6-4\beta-\sqrt{2\beta})}{4(2-\beta)}, & \text{otherwise} \end{cases}$$

Clearly then, the retailer is better off in the presence of piracy or its threat if $\rho_3 < r < \rho_4$. The three regions in the theorem then emerge by combining the above. ■

Proof of Proposition 3

The consumer surplus (CS) for all consumers, legal and illegal, can be found by aggregating their consumption benefits net of the price they pay or the penalty they incur. Therefore, CS is given by

$$CS = \begin{cases} \int_{\frac{p^*-r}{1-\beta}}^{1} (v - p^*)dv + \underbrace{\int_{\frac{r}{\beta}}^{\frac{p^*-r}{1-\beta}} (\beta v - r)dv}_{\text{Pirate Surplus}}, & \text{if } p^* \geq \frac{r}{\beta} \\ \int_{p^*}^{1} (v - p^*)dv, & \text{otherwise} \end{cases}$$

The desired result can now be obtained by algebraic manipulation after substituting p^* from (2) into the expression above.

The above expression includes the net surplus from the legal users as well as that from the pirates. If one is interested in finding the consumer surplus excluding that of the pirates, it can be easily accomplished by dropping the term marked as "Pirate Surplus" above. ■

Proof of Theorem 2

In the piracy region, $CS = \frac{1+15\beta-30r}{32} + \frac{r^2}{32}\left(\frac{1}{1-\beta} + \frac{16}{\beta}\right)$. Its derivative, $\frac{\partial(CS)}{\partial r} = r\left(\frac{1}{16(1-\beta)} + \frac{1}{\beta}\right) - \frac{15}{16}$ is an increasing function of r. However, since $r < \rho_1$ in the piracy region, we must have

$$\frac{\partial(CS)}{\partial r} < \rho_1\left(\frac{1}{16(1-\beta)} + \frac{1}{\beta}\right) - \frac{15}{16} = -\frac{3}{4(4-3\beta)} < 0$$

In other words, in the piracy region, CS is decreasing in r, and is minimized at $r = \rho_1$. Now, $CS|_{r=\rho_1} = \frac{1}{2(4-3\beta)^2} > \frac{1}{32}$. Clearly then, CS in the piracy region is always above the benchmark value of $CS_0 = \frac{1}{32}$.

Furthermore, the consumer surplus in the threat region, $CS = \frac{(\beta-r)^2}{2\beta^2}$ is decreasing in r. Therefore, by equating it to CS_0, we find that consumers are better off if $r < \rho_5 = \frac{3\beta}{4}$.

Since $\rho_4 < \rho_5$ for all $\beta > 0$, the result follows from Theorem 1. ■

Proof of Proposition 4

Since the channel profit is given by $CP = \pi_m^* + \pi_r^*$, it can be easily calculated from Proposition 2. Further, social welfare can be calculated from

$$SW = \begin{cases} \displaystyle\int_{\frac{p^*-r}{1-\beta}}^{1} v\,dv + \underbrace{\int_{\frac{r}{\beta}}^{\frac{p^*-r}{1-\beta}} \beta v\,dv}_{\text{Welfare from Piracy}}, & \text{if } p^* \geq \frac{r}{\beta} \\ \displaystyle\int_{p^*}^{1} v\,dv, & \text{otherwise} \end{cases}$$

Substituting p^* from (2) into the above expression, we get the desired result. Of course, if one is interested in calculating the social surplus without including the pirates, it can be easily done by dropping the term labeled "Welfare from Piracy" above. ■

Proof of Theorem 3

In the piracy region, $CP = \frac{3(1-\beta+r)^2}{16(1-\beta)}$ is increasing in r. Equating it to CP_0, we get $r = \sqrt{1-\beta} - (1-\beta)$, which is valid only if it is less than ρ_1, or equivalently, if $\beta < \frac{8}{9}$.

Now, in the threat region, $CP = \frac{r(\beta-r)}{\beta^2}$ is concave in r. Equating it to CP_0, we get two roots, $r = \rho_5^c = \frac{\beta}{4}$ and $r = \rho_5 = \frac{3\beta}{4}$. The first root is less than ρ_1 and the second greater; as long as r is between these two roots, CP is higher than its benchmark. Defining

$$\rho_6 = \begin{cases} \sqrt{1-\beta} - (1-\beta), & \text{if } \beta \leq \frac{8}{9} \\ \dfrac{\beta}{4}, & \text{otherwise} \end{cases}$$

we conclude that channel profit is higher if $\rho_6 < r < \rho_5$.

As far as social welfare is concerned, in the piracy region, $SW = \frac{7+9\beta+6r}{32} - \frac{r^2}{32}\left(\frac{1}{1-\beta} + \frac{16}{\beta}\right)$ is clearly concave in r. Therefore, the minimum value of SW occurs at one of the extremes, that is either at $r = 0$ or at $r = \rho_1$. Both these extreme values of SW can be easily shown to be greater than SW_0, implying that piracy always leads to a higher social surplus. We now move to the threat region, where $SW = \frac{1}{2}\left(1 - \frac{r^2}{\beta^2}\right)$ is clearly decreasing in r. Equating it to SW_0, we find that the threat region does better in terms of social welfare, if $r < \rho_5 = \frac{3\beta}{4}$. This completes the proof. ■

Proof of Proposition 5

When $\bar{p} > \frac{r}{\beta}$, $\bar{\pi} = \bar{p}q(\bar{p}) = \bar{p}\left(1 - \frac{\bar{p}-r}{1-\beta}\right)$, implying $\frac{\partial\bar{\pi}}{\partial\bar{p}} = 1 - \frac{2\bar{p}-r}{1-\beta}$.

Since $\frac{\partial^2\bar{\pi}}{\partial\bar{p}^2} = -\frac{2}{1-\beta} < 0$, the first order condition, $\frac{\partial\bar{\pi}}{\partial\bar{p}} = 0$, resuls in $\bar{p}^* = \frac{1}{2}(1-\beta+r)$. Clearly, this solution must be greater than $\frac{r}{\beta}$, or $r < \frac{(1-\beta)\beta}{2-\beta} = \bar{\rho}_1$.

If, on the other hand, $\bar{p} < \frac{r}{\beta}$, then $\bar{\pi} = \bar{p}q(\bar{p}) = \bar{p}(1 - \bar{p})$, resulting in $\bar{p}^* = \frac{1}{2}$. This $\bar{p}^*$ should be less than or equal to $\frac{r}{\beta}$, implying $r \geq \frac{\beta}{2} = \bar{\rho}_2$.

Now, $\bar{\rho}_2 - \bar{\rho}_1 = \frac{\beta^2}{2(2-\beta)}$, that is, $\bar{\rho}_1 < \bar{\rho}_2$. Therefore, we also consider the situation where $\bar{\rho}_1 \leq r < \bar{\rho}_2$. In that situation, the profit above is decreasing for $\bar{p} > \frac{r}{\beta}$ but increasing for $\bar{p} < \frac{r}{\beta}$. So, $\bar{p}^*$ becomes $\frac{r}{\beta}$.

The optimal profit in each region can be found easily from $\bar{p}^* q(\bar{p}^*)$. ■

Proof of Theorem 4

We start by noting that, in the benchmark region, where neither piracy nor its threat is present, $\eta = \frac{3/16}{1/4} = \frac{3}{4}$. We will now show that, for $\bar{\rho}_1 < r < \rho_5, \eta$ is larger than $\frac{3}{4}$. To do so, we make use of Proposition 4 and Proposition 5. This allows us to divide the interval $(\bar{\rho}_1, \rho_5)$ into several parts:

- When $\bar{\rho}_1 < r < \rho_1$, $CP = \frac{3(1-\beta+r)^2}{16(1-\beta)}$ and $\bar{\pi}^* = \frac{r(\beta-r)}{\beta^2}$. Therefore, $\eta = \frac{3\beta^2(1-\beta+r)^2}{16r(1-\beta)(\beta-r)}$, and $\eta - \frac{3}{4} = \frac{3\left(\beta(1-\beta)-r(2-\beta)\right)^2}{16r(1-\beta)(\beta-r)} > 0$.
- If $\rho_1 \leq r \leq \bar{\rho}_2$, $CP = \bar{\pi}^* = \frac{r(\beta-r)}{\beta^2}$. Therefore, $\eta = 1$, and the channel is fully coordinated.
- Finally, when $\bar{\rho}_2 < r < \rho_5$, $CP = \frac{r(\beta-r)}{\beta^2}$ and $\bar{\pi}^* = \frac{1}{4}$. Therefore, $\eta = \frac{4r(\beta-r)}{\beta^2}$, which is greater than $\frac{3}{4}$ because $r < \rho_5 = \frac{3\beta}{4}$. ■

Proof of Lemma 2

It is easy to verify that the manufacturer's profit is increasing in r in the piracy region, but concave in the threat region, implying that the maximum must happen in the threat region. The manufacturer's profit in the threat region is given by

$$\pi_m^* = \frac{(\beta - r)(r - (1-\beta)(\beta - r))}{\beta^2}$$

Since $\frac{\partial^2 \pi_m^*}{\partial r^2} = -\frac{2(2-\beta)}{\beta^2} < 0$, we simply solve $\frac{\partial \pi_m^*}{\partial r} = 0$ to obtain $r_m^* = \frac{\beta(3-2\beta)}{4-3\beta}$.

As far as the retailer is concerned, it can be easily verified that its profit is increasing in r in the piracy region and decreasing in the threat region. Therefore, it is maximized at ρ_1, implying $r_r^* = \rho_1 = \frac{3\beta(1-\beta)}{4-3\beta}$. Of course, the profit at r_r^* can be better than the benchmark profit only if $\beta < \frac{8}{9}$. If $\beta \geq \frac{8}{9}$, however, the retailer would prefer an r that is greater than ρ_2.

It is also easy to verify that the channel profit is increasing in r in the piracy region, but concave in the threat region. The channel profit, *CP*, in the threat region is $\frac{r(\beta-r)}{\beta^2}$, which is maximized at $r_c^* = \frac{\beta}{2}$.

Consumer surplus is always decreasing in r, implying that the maximum occurs at r_C^*.

Finally, the total social welfare, SW, is concave in r in the piracy region, but decreasing in the threat region. Now, in the piracy region

$$SW = \frac{7 + 9\beta + 6r}{32} - \frac{r^2}{32}\left(\frac{1}{1-\beta} + \frac{16}{\beta}\right)$$

Since $\frac{\partial^2(SW)}{\partial r^2} = -\left(\frac{1}{16(1-\beta)} + \frac{1}{\beta}\right) < 0$, we can solve $\frac{\partial(SW)}{\partial r} = 0$ to get $r_S^* = \frac{3\beta(1-\beta)}{16-15\beta}$. ■

Proof of Proposition 6

To prove this result, we need to show that $r_C^*, r_S^* < r_r^*, r_c^*, r_m^*$. Now it can be easily shown that $r_r^* < r_c^* < r_m^*$. Further, because $r_C^* = 0$, $r_C^* < r_S^*$ holds trivially. Therefore the proof can be completed by simply showing that $r_S^* < r_r^*$. Now,

$$r_S^* - r_r^* = \frac{\beta(36 - 59\beta + 24\beta^2)}{64 - 92\beta + 30\beta^2} > 0, \forall \beta \in (0,1)$$

which completes the proof. ■